

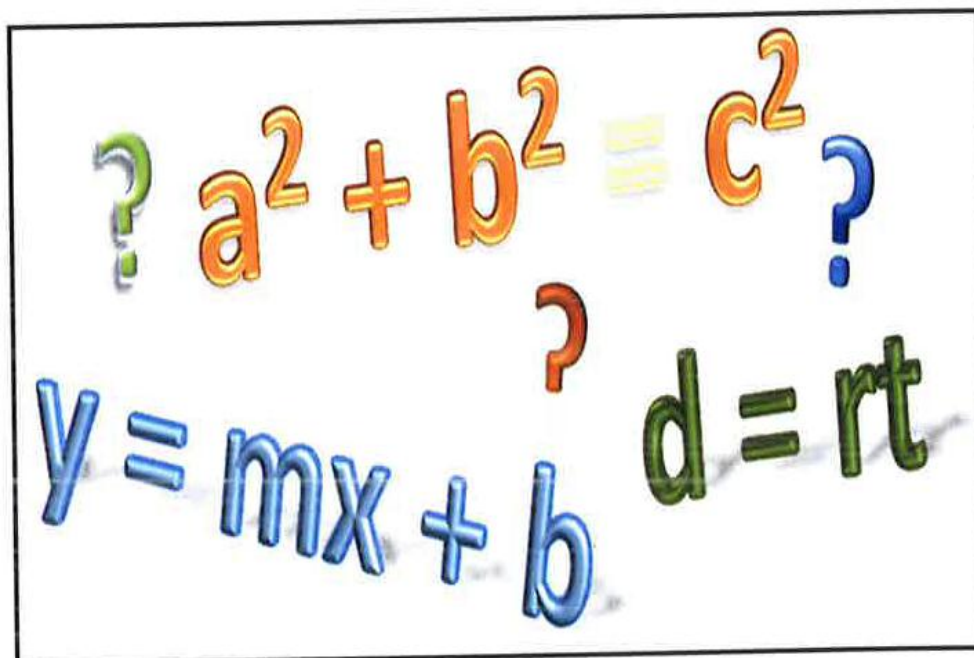


DEPARTMENT OF EDUCATION

GRADE 9

MATHEMATICS

UNIT 2



PATTERNS AND CHANGE

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FOR DEPARTMENT OF EDUCATION
PAPUA NEW GUINEA

GRADE 9

MATHEMATICS

UNIT 2

TOPIC 1: DIRECTED NUMBERS

TOPIC 2: INDICES

TOPIC 3: ALGEBRAIC EXPRESSIONS

TOPIC 4: EQUATIONS

Acknowledgements

We acknowledge the contribution of all Secondary and Upper Primary teachers who in one way or another helped to develop this Course.

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MR. DEMAS TONGOGO
Principal-FODE



Flexible Open and Distance Education
Papua New Guinea

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Papua New Guinea

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Written by: Mrs. Estrella V. Tuguinay

Compiled and finalised by: Mrs. Luzviminda B. Fernandez
SCO -Mathematics Department-FODE

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SECRETARY'S MESSAGE

Achieving a better future by individuals students, their families, communities or the nation as a whole, depends on the curriculum and the way it is delivered.

This course is part and parcel of the new reformed curriculum – the Outcome Base Education (OBE). Its learning outcomes are student centred and written in terms that allow them to be demonstrated, assessed and measured.

It maintains the rationale, goals, aims and principles of the National OBE Curriculum and identifies the knowledge, skills, attitudes and values that students should achieve.

This is a provision of Flexible, Open and Distance Education as an alternative pathway of formal education.

The Course promotes Papua New Guinea values and beliefs which are found in our constitution, Government policies and reports. It is developed in line with the National Education Plan (2005 – 2014) and addresses an increase in the number of school leavers which has been coupled with a limited access to secondary and higher educational institutions.

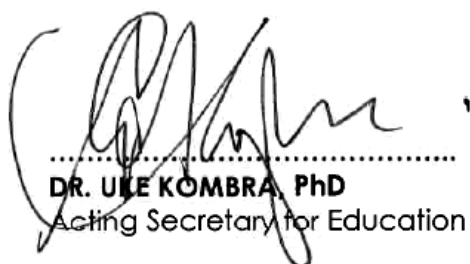
Flexible, Open and Distance Education is guided by the Department of Education's Mission which is fivefold;

- to facilitate and promote integral development of every individual
- to develop and encourage an education system which satisfies the requirements of Papua New Guinea and its people
- to establish, preserve, and improve standards of education throughout Papua New Guinea
- to make the benefits of such education available as widely as possible to all of the people
- to make education accessible to the physically, mentally and socially handicapped as well as to those who are educationally disadvantaged

The College is enhanced to provide alternative and comparable path ways for students and adults to complete their education, through one system, many path ways and same learning outcomes.

It is our vision that Papua New Guineans harness all appropriate and affordable technologies to pursue this program.

I commend all those teachers, curriculum writers and instructional designers, who have contributed so much in developing this course.



.....
DR. UKE KOMBRA, PhD
Acting Secretary for Education

UNIT 2: PATTERNS OF CHANGE



Dear Student

This is the second unit of the Grade 9 Mathematics Course. This unit is based on the NDOE Lower Secondary Syllabus and Curriculum Framework for Grade 9.

This Unit consists of four topics:

- Topic 1: Directed Numbers**
- Topic 2: Indices**
- Topic 3: Algebraic Expressions**
- Topic 4: Equations**

In Topic 1-**Directed Numbers**- You will revise directed numbers, then present directed numbers on number lines and relate them to real life situations. You will also learn to perform addition, subtraction, multiplication and division of directed numbers on practical problems in real life situations

In Topic 2- **Indices**- You will revise squares and square roots and learn to use the table of squares and square roots to find the squares and square root of a given number. You will also learn the different Laws of indices and solve problems involving Scientific notation.

In Topic 3- **Algebraic Expressions**- You will learn about number patterns. You will also learn translate words, phrases and sentences in to algebraic expressions, then simplify, evaluate and factorize algebraic expressions.

In Topic 4- **Equations**- You will define simple equations and identify the properties and the steps in solving simple equations. You will also learn to solve equations involving grouping symbols, fractions and unknown on both sides as well as changing the subject and transposition of formulae. Then, solve word problems using equations.

You will find that each lesson has reading material to study, worked examples and a practice exercise. The answers to the practice exercises are given at the end of each topic.

All the lessons are written in simple language with comic characters to guide you. The practice exercises are graded to help you learn the process of working out problems.

We hope you enjoy learning this unit.

Mathematics Department
FODE

STUDY GUIDE

Follow the steps given below as you work through the Unit.

- Step 1: Start with TOPIC 1 Lesson 1 and work through it.
- Step 2: When you complete Lesson 1, do Practice Exercise 1.
- Step 3: After you have completed Practice Exercise 1, check your work. The answers are given at the end of TOPIC 1.
- Step 4: Then, revise Lesson 1 and correct your mistakes, if any.
- Step 5: When you have completed all these steps, tick the check-box for the Lesson, on the Contents Page (page 3) like this:



Lesson 1: Directed numbers in Practical Situations

Then go on to the next Lesson. Repeat the process until you complete all of the lessons in Topic 1.

As you complete each lesson, tick the check-box for that lesson, on the Content Page 3, like this ☒. This helps you to check on your progress.

- Step 6: Revise the Topic using Topic 1 Summary, then, do Topic test 1 in Assignment 2.

Then go on to the next Topic. Repeat the same process until you complete all of the four Topics in Unit 2.

Assignment: (Four Topics and a Unit Test)

When you have revised each Topic using the Topic Summary, do the Topic Test in your Assignment. The Unit book tells you when to do each Topic Test.

When you have completed the four Sub-strand Tests, revise well and do the Strand test. The Assignment tells you when to do the Strand Test.

The Topic Tests and the Unit test in the Assignment will be marked by your Distance Teacher. The marks you score in each Assignment will count towards your final mark. If you score less than 50%, you will repeat that Assignment.

Remember, if you score less than 50% in three Assignments, you will not be allowed to continue. So, work carefully and make sure that you pass all of the Assignments.

TOPIC 1

DIRECTED NUMBERS

Lesson 1:	Directed Numbers in Practical Situations
Lesson 2:	Adding Directed Numbers
Lesson 3:	Subtracting Directed Numbers
Lesson 4:	Multiplying Directed Numbers
Lesson 5:	Dividing Directed Numbers
Lesson 6:	Solving Mixed Problems

TOPIC 1: DIRECTED NUMBERS

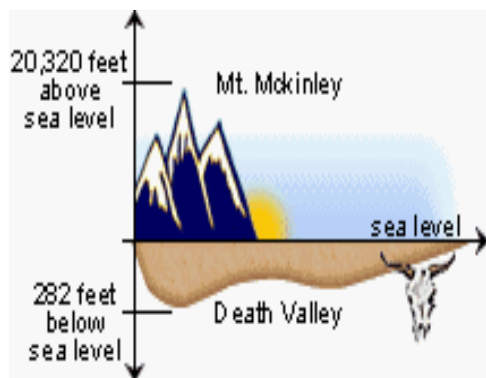
Introduction



Welcome to Topic 1 Unit 2 of Grade 9 Mathematics.

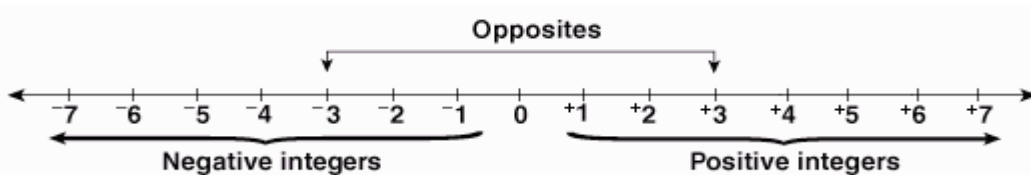
Problem: The highest elevation in North America is Mt. McKinley, which is 20,320 feet above sea level. The lowest elevation is Death Valley, which is 282 feet below sea level. What is the distance from the top of Mt. McKinley to the bottom of Death Valley?

The problem above uses the notion of **opposites**: Above sea level is the opposite of below sea level. Here are some more examples of opposites:



top, bottom increase, decrease forward, backward positive, negative

We could solve the problem above using **DIRECTED NUMBERS**. Directed numbers are the set of whole numbers and their opposites. These numbers are also called **integers**. The number line is used to represent integers. This is shown below.



Two integers are opposites if they are each the same distance away from zero, but on opposite sides of the number line. One will have a positive sign, the other a negative sign. In the number line above, +3 and -3 are labelled as opposites.

Definitions

- The number line goes on forever in opposite directions. This is indicated by the arrows.
- Whole numbers greater than zero are called positive integers. These numbers are to the right of zero on the number line. Whole numbers less than zero are called negative integers. These numbers are to the left of zero on the number line. The integer zero is neutral. It is neither positive nor negative.
- The sign of an integer is either positive (+) or negative (-), except zero, which has no sign.
- Two integers are opposites if they are each the same distance away from zero, but on opposite sides of the number line. One will have a positive sign, the other a negative sign. In the number line shown above, +3 and -3 are labeled as opposites.

Lesson 1: Directed Numbers in Practical Situations



You learnt something about directed numbers in Upper Primary Mathematics.



In this lesson, you will:

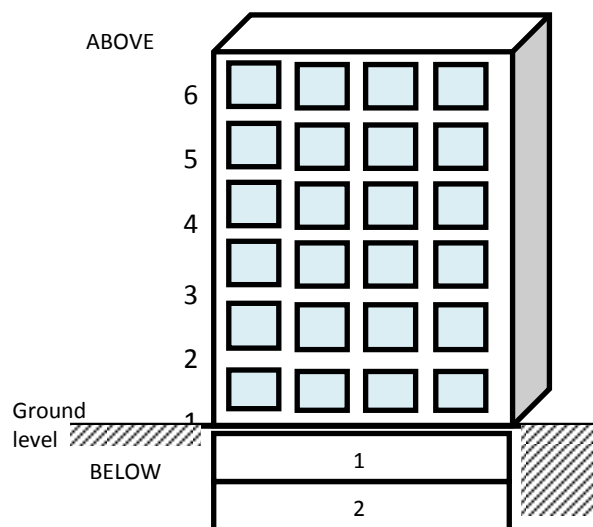
- define directed numbers
- present directed numbers on number lines
- identify directed numbers in real situations

The building on the right hand side of the page has 6 floors above the ground level and 2 floors below the ground level. The following information is given about the people living in the building:

“Jessica lives on the fourth floor of the building”

“Peter parked his car on the second floor below the ground level of the building.”

In the above statements, words are used to tell the **number** of the floor the person is in.



However, the words also give us the **direction** of the floor, meaning we can tell whether the floor is above or below the ground level.

In mathematics, words are often replaced with numbers to help us describe the direction of a number, we use a set of numbers called **directed numbers**.

Directed numbers are positive and negative numbers that give the distance and direction of a number. This set of directed numbers are also called the set of integers.

Directed numbers can be divided into two categories: the **Positive numbers** and **Negative Numbers**.

Positive numbers are numbers that are **greater** than zero. They are written with a plus (+) sign in front of the number.

Here are some examples of positive numbers: +5, +10, +23 and so on.

Positive numbers are also used to represent an increase or a gain in a practical life situation.

See example on the next page.

Example 1

Jack gained 5 kilos

In this situation, there has been a gain in weight and so the directed number suggested by the situation is **+5**.

Example 2

This fortnight, Peter earned an extra K100 in his salary..

In this situation, Peter's salary has increased. So the directed number suggested by the situation is **+K100**.

Negative numbers are numbers that are **less** than zero. They are written with a minus (-) sign in front of the number.

Some examples of negative numbers are: -10, -4, -28, and so on.

Negative numbers are also used to represent a decrease or a loss in a practical life situation.

Example 1

Sarah's new office is two floors down from her old office.

In this situation, there has been a decrease in floor levels and so the directed number suggested by the situation is **-2**.

Example 2

The temperature in Sydney last winter was 3°C below zero.

In this situation, the temperature is below zero. So the directed number suggested by the situation is **-3°C**.

Positive numbers and negative numbers are opposites of each other.

So for a positive number or a statement that describes a situation with an increase or gain, there is an equivalent negative number or statement describing a decrease or a loss.

Example 1

- a. The opposite of +5 is -5
- b. The opposite of -100 is +100.
- c. The opposite of +45 is -45.
- d. The opposite of 5 cm to the Right is 5 cm to the Left
- e. The opposite of 8m below ground level is 8 m above ground level
- f. If John **won** K10 credits, then the opposite of this statement is that John **lost** K10.

Example 2

Write an integer to represent each situation:

10 degrees above zero	+10
a loss of 16 dollars	-16
a gain of 5 points	+5
8 steps backward	-8

Example 3

Name the opposite of each integer.

Integer	Opposite
-12	+12
+21	-21
-17	+17
+9	-9

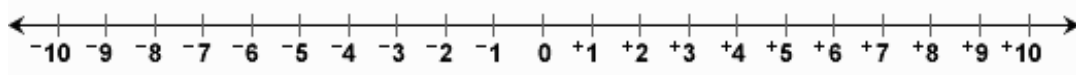
Example 4

Name 4 real life situations in which integers can be used.

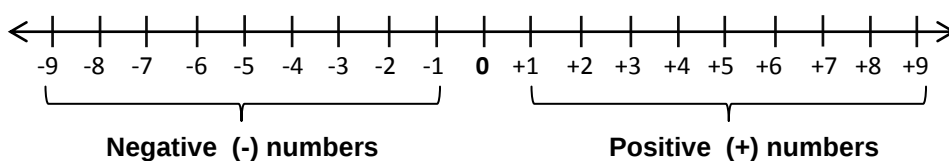
- Spending and earning money.
- Rising and falling temperatures.
- Stock market gains and losses.
- Gaining and losing yards in a football game.

The Number Line

An visual and useful way of dealing with directed numbers is by using the **number line** since the positions of positive and negative numbers can be clearly seen. An example of a number line is shown below:



The middle point of the number line is zero, 0. The positive numbers are found on the right side of zero while the negative numbers are on the left side of zero. So:

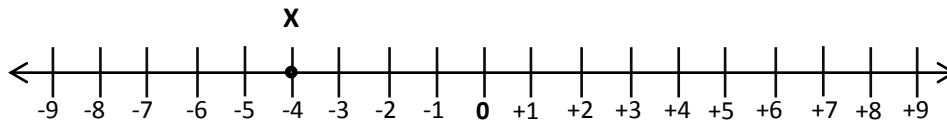


Here are other examples using a number line to present directed numbers.

Example 1:

Draw a number line between -9 and +9. Using the letter X, create a mark at -4.

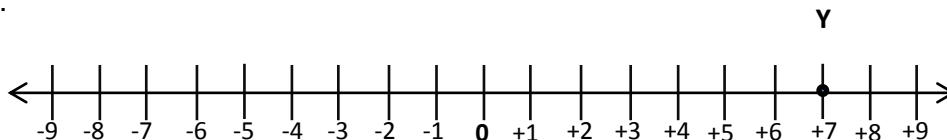
Solution :



Example 2:

Draw a number line between -9 and +9. Using the letter Y, create a mark at +7.

Solution:



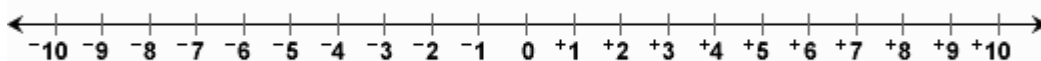
Note:

A positive integer does not have to have a + sign in it. For example, +3 and 3 are the same .

Comparing Directed Numbers

The bigger the number, the further to the right it will be on the number line.

The number line is also helpful when we are comparing directed numbers.



Therefore, since positive numbers are on the right side of the number line we can say that:

Positive numbers are numbers greater than negative numbers.

When comparing two integers, we use the symbols $<$ and $>$. (Note: The symbol always points to the smaller number.)

- | | |
|-----------|-----------------------------|
| $1 < 5$ | means 1 is less than 5 |
| $-7 < -5$ | means -7 is less than -5 |
| $+9 > -6$ | means +9 is greater than -6 |
| $-2 > -4$ | means -2 is greater than -4 |

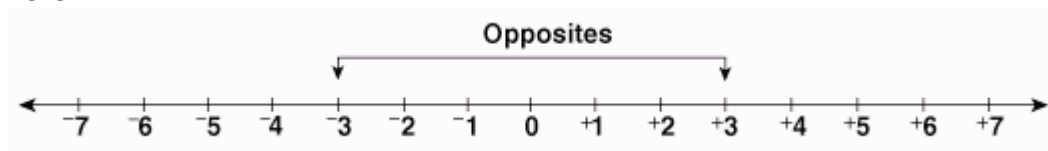
Instead of using "greater than" and "less than" to compare directed numbers, we can use symbols.

$>$ means greater than
 $<$ means less than



What is an ABSOLUTE VALUE?**Absolute Value**

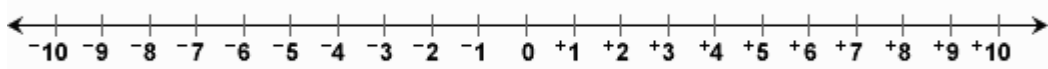
Two integers that are the same distance from zero in opposite directions are called **opposites**. The integers +3 and -3 are opposites since they are each 3 units from zero.



The absolute value of an integer is its distance from zero on the number line.

The absolute value of +3 is 3, and the absolute value of -3 is 3. Thus, opposite integers have the same absolute value. Mathematicians use the symbol $| |$ for absolute value. Let's look at some examples.

Example 1: Find the absolute value of +3, -3, +7, -5, +9, -8, +4, -4. You may refer to the number line below.



Solution:

$$|+3| = 3$$

$$|+9| = 9$$

$$|-3| = 3$$

$$|-8| = 8$$

NOW DO PRACTICE EXERCISE 1

**Practice Exercise 1**

1 Write down a directed number suggested by each of the following:

- | | |
|---------------------------|-----------------------|
| a.) 3m above the surface | d.) A loss by 4 goals |
| b.) A weight loss of 5 kg | e.) Up to 28 floors |
| c.) I owe the bank K250 | |
-

2 Copy and complete

- a.) If north is a positive direction, then south is a _____ direction.
b.) If right is a positive direction, then _____ is a _____ direction.
-

3 State the opposite:

- | | |
|--------------------|---------------------|
| a.) Minus 7 | d.) 50m underground |
| b.) Down 100 steps | e.) 3 days early |
| c.) South 20k | |
-

4 Write the correct symbol (< or >) between each pair of values below.

- | | |
|-----------------|------------------|
| a.) -9 _____ -4 | d.) 13 _____ -13 |
| b.) 0 _____ -10 | e.) -15 _____ 1 |
| c.) 7 _____ -48 | |
-

5 The minimum temperatures recorded at Ontario , Canada for one week were: Monday -3°C, Tuesday -7°C, Wednesday 0°C, Thursday -1°C, Friday 2°C, Saturday -2°C and Sunday -4°C

- a.) On which day was the lowest minimum temperature?
b.) When was the highest minimum recorded?
-

6 Find the absolute value of the following : 11 , -9 , 14 , -10, and -20.

7 Write these integers below from

- a.) least to greatest -9, +3, -23, +6, -7
b.) greatest to least +8, -13, -19, 0, +11, -15
-

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 2

Lesson 2: Adding Directed Numbers



You learnt to identify directed numbers in real life situation in the previous lesson..



In this lesson, you will:

- add directed numbers
- solve applied problems on directed numbers.

Addition of Directed Numbers

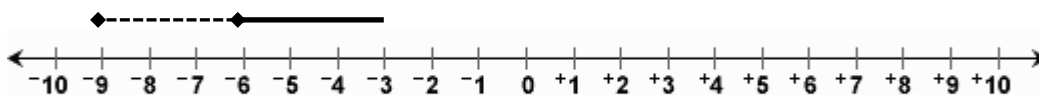
Problem: George owes his friend Jeanne K3. If he borrows another K6, how much will he owe her altogether?

Solution: This problem is quite simple: just add K3 and K6 and the result is K9.

The problem above can be solved using addition of integers. Owing K3 can be represented by -3 and owing another K6 can be represented by -6. The problem becomes:

$$-3 + -6 = -9$$

Look at the number line below. If we start at 0, and move 3 to the left, we land on -3. If we then move another 6 to the left, we end up at -9.



Addition of Like Signs

Rule # 1: The sum of two negative integers is a negative integer

Example 1: Find the sum of each pair of integers. You may draw a number line to help you solve this problem.

Adding Negative Integers	
Integers	Sum
$-2 + -9 =$	-11
$-5 + -8 =$	-13
$-13 + -7 =$	-20

NOTE: Do not confuse the sign of the integer with the operation being performed.

Remember that:

$-2 + -9 = -11$ is read as Negative 2 plus negative 9 equals negative 11.

Rule # 2: The sum of two positive integers is a positive integer.

Example 2:

Find the sum of each pair of integers. You may draw a number line to help you solve this problem.

Adding Positive Integers	
Integers	Sum
$2 + 7 =$	9
$5 + 8 =$	13
$13 + 7 =$	20

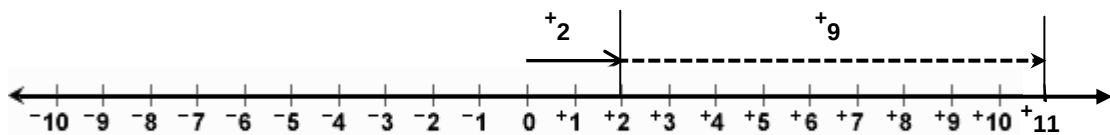
NOTE: Do not confuse the sign of the integer with the operation being performed.

Remember that:

$29 + 16 = 45$ is read as Positive 29 plus positive 16 equals positive 45.

Using the number line, it will look this way:

$2 + 9 = 11$ (Both are positive, so the directions of both arrows will be to the right)



Addition of Unlike Signs

So far we have added integers with like signs. What happens when we add integers with unlike signs? How do we add a positive and a negative integer, or a negative and a positive integer?

Procedure: To add a positive and a negative integer (or a negative and a positive integer), follow these steps:

- 1) Find the absolute value of each integer.
- 2) Subtract the smaller number from the larger number you get in Step 1.
- 3) The result from Step 2 takes the sign of the integer with the greater absolute value.

We will use the above procedure to add integers with unlike signs in Examples 3 through 6. Refer to the number line to help you visualize the process in each example. We will use money as an alternative method for adding integers.

Example 3: Find the sum of +7 and -4.

Step 1: $|+7| = 7$ and $|-4| = 4$

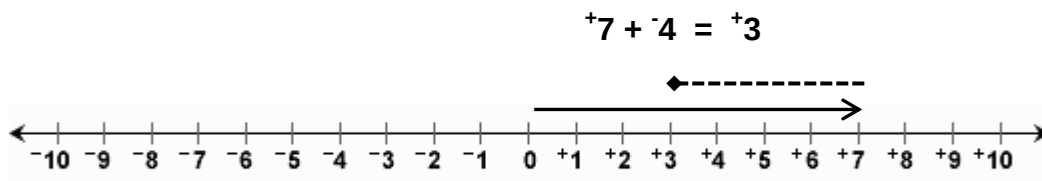
Step 2: $7 - 4 = 3$

Step 3: The number 3 will take a positive sign since +7 is farther from zero than -4.

Solution 1: $+7 + -4 = +3$

Solution 2: If you start with K7 and you owe K4, then you end up with K3.

Solution 3: Using the number line, show where the arrows will stop.



Example 4: Find the sum of -9 and +5.

Step 1: $|-9| = 9$ and $|+5| = 5$

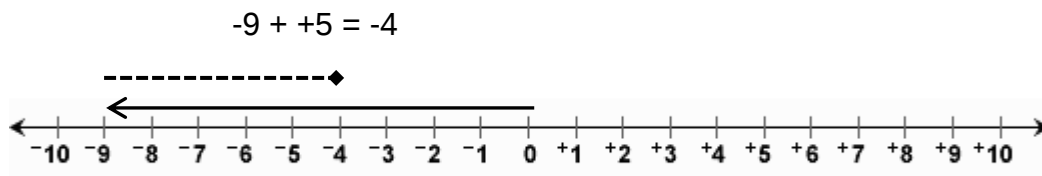
Step 2: $9 - 5 = 4$

Step 3: The number 4 will take a negative sign since -9 is farther from 0 than +5.

Solution 1: $-9 + +5 = -4$

Solution 2: If you are owed K9 and you are paid K5, then you are still short K4.

Solution 3: Using the number line, show where the arrows will stop.



Example 5: Find the sum of +6 and -7.

Step 1: $|+6| = 6$ and $|-7| = 7$

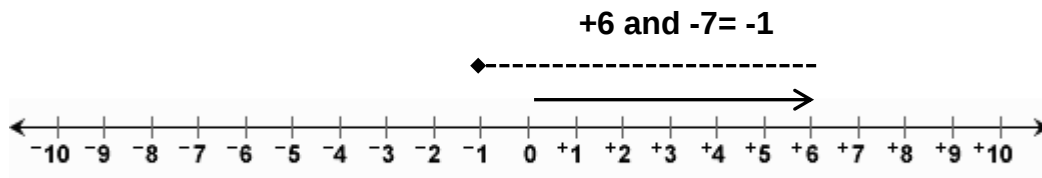
Step 2: $7 - 6 = 1$

Step 3: The number 1 will take a negative sign since -7 is farther from 0 than +6.

Solution 1: $+6 + -7 = -1$

Solution 2: If you start with K6 and you owe K7, then you are still short of K1.

Solution 3: Using the number line, show where the arrows will stop.



Example 6: Find the sum of +9 and -9.

Step 1: $|+9| = 9$ and $|-9| = 9$

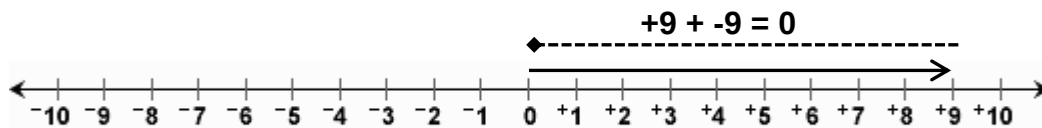
Step 2: $9 - 9 = 0$

Step 3: The integer 0 has no sign.

Solution 1: $+9 + -9 = 0$

Solution 2: If you start with K9 and you owe K9, then you end up with K0.

Solution 3:



In Example 6 you will notice that the integers +9 and -9 are opposites. Look at the problems below. Do you see a pattern?

$$^{-}100 + ^{+}100 = 0$$

$$^{+}349 + ^{-}349 = 0$$

$$^{-}798 + ^{+}798 = 0$$

Rule # 3 : The sum of any integer and its opposite is equal to zero.

Summary:

- Adding two positive integers always yields a positive sum; adding two negative integers always yields a negative sum.
- To find the sum of a positive and a negative integer, take the absolute value of each integer and then subtract these values. The result takes the sign of the integer with the larger absolute value.
- The sum of any integer and its opposite is equal to zero.

NOW DO PRACTICE EXERCISE 2

**Practice Exercise 2**

1. Use a number line to find the finishing point for each of the following trips:

- a) $2 + (-6)$
 - b) $0 + (+4)$
 - c) $+7 + (-7)$
 - d) $-8 + (+12)$
 - e) $-6 + (-6)$
-

2. Evaluate:

- a) $+10 + (+17)$
 - b) $-8 + (+14)$
 - c) $-6 + (-80)$
 - d) $-3 + (+11)$
 - e) $-63 + (+99)$
 - f) $35 + (-20)$
-

3. The temperature in the desert is -12°C , but rises by 20 degrees. What is the new temperature?

4. A lift is at the third basement floor (3 floors below ground level), and moves up 14 levels. On which level does it end up?

5. A diver jumps from a platform 12 m high and dives 17 m to the bottom of a pool. How deep is the pool?

6. You have K5 in your pocket, but you owe a friend K12 while someone else owes you K20. If everyone pays up, how much will you have at the end?

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 1

Lesson 3: Subtracting Directed Numbers



You learnt to add directed numbers and their applications in real life situations in Lesson 2.



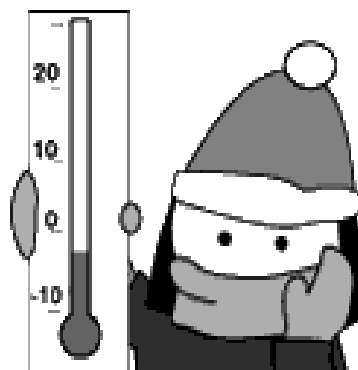
In this lesson, you will:

- subtract directed numbers
- solve applied problems on directed numbers.

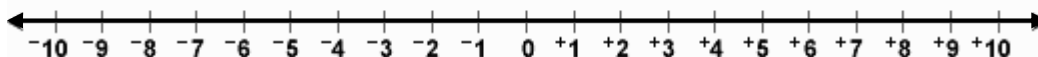
Subtraction of Directed Numbers

Problem: The temperature in Anchorage, Alaska was 8°F in the morning and dropped to -5°F in the evening. What is the difference between these temperatures?

Solution: We can solve this problem using integers. Using the number line below, the distance from $+8$ to 0 is 8 , and the distance from 0 to -5 is 5 , for a total of 13 .



$+8 - (-5) = +13$. The difference is 13 degrees.



We need a rule in subtracting integers in order to solve the above problem.

In the above problem, we added the opposite of the second integer and subtraction was transformed into addition. Let's look at some simpler examples of subtracting integers.

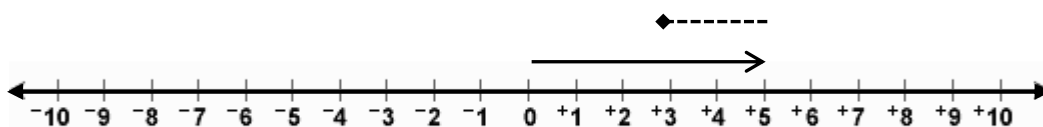
Example 1: $+5 - (+2)$

Step 1: The opposite of $+2$ is -2 .

Step 2: Subtraction becomes addition.

Solution: $+5 - (+2) = +5 + (-2) = +3$

Using the number line : $+5 - (+2) = +5 + (-2) = +3$



Example 2: Find the difference between each pair of integers.

Subtracting Integers		
Subtract	Add the opposite	Result
$+6 - (+2) =$	$6 + (-2) =$	4
$+6 - (-2) =$	$6 + 2 =$	8
$-6 - (+2) =$	$-6 + (-2) =$	-8
$-6 - (-2) =$	$-6 + 2 =$	-4

Notice that in each problem, the first integer remained unchanged. Also do not confuse the sign of the operation and the sign of the integer with the operation being performed.

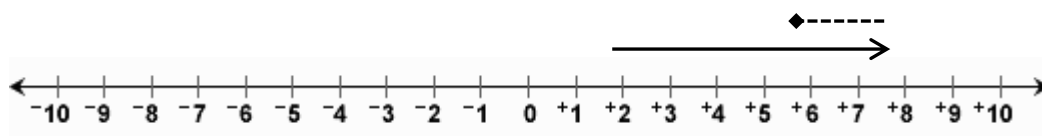
Remember that:

$-9 + (4) = -5$ is read as Negative 9 plus positive 4 equals negative 5

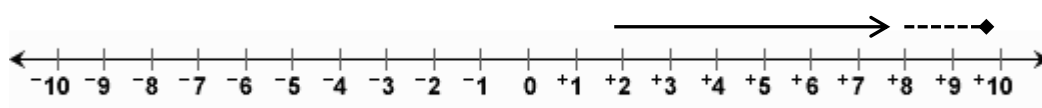
The number line can also be used to find the solution. Look at the illustration below:

Solutions:

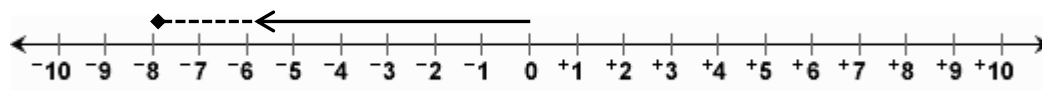
$+6 - (+2) =$	$6 - 2 =$	4
---------------	-----------	---



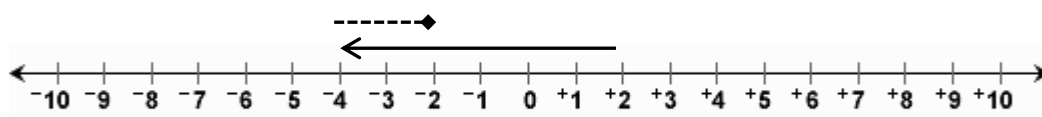
$+6 - (-2) =$	$6 + 2 =$	8
---------------	-----------	---



$-6 - (+2) =$	$-6 + (-2) =$	-8
---------------	---------------	----



$-6 - (-2) =$	$-6 + 2 =$	-4
---------------	------------	----



Let us look at more examples.

Example 3

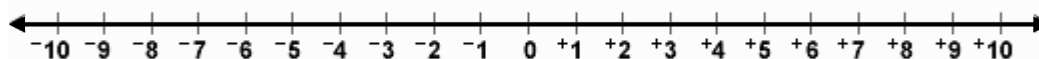
Find the difference between each pair of integers.

Subtracting Integers		
Subtract	Add the opposite	Result
$+7 - (+10) =$	$+7 + (-10) =$	-3
$+7 - (-10) =$	$+7 + 10 =$	$+17$
$-7 - (+10) =$	$-7 + (-10) =$	-17
$-7 - (-10) =$	$-7 + 10 =$	$+3$

Example 4:

Find the difference between each pair of integers. You may extend the number line below to help you solve these problems.

Subtracting Integers		
Subtract	Add the opposite	Result
$-8 - (3) =$	$-8 + (-3) =$	-11
$+17 - (-9) =$	$+17 + 9 =$	$+26$
$-12 - (15) =$	$-12 + (-15) =$	-27
$-19 - (-23) =$	$-19 + 23 =$	$+4$



With all the given examples so far, we have come to this rule.

To subtract an integer, add its opposite.

When subtracting integers, it is important to show all work, as we did in this lesson. If you skip steps, or do the work in your head, you are very likely to make a mistake--even if you are a top math student!

NOW DO PRACTICE EXERCISE 3

**Practice Exercise 3**

1. Complete the table. Use the rule to subtract the following:

Subtract	Add the opposite	Result
a) $8 - 10$		
b) $4 - 5$		
c) $-2 - (-6)$		
d) $10 - 7$		

2. Evaluate:

- a) $+10 - (+18)$
 - b) $-36 - (+4)$
 - c) $-7 - (-37)$
 - d) $-66 - (+87)$
 - e) $-5 - (+13)$
-

3. The temperature at sunset was 12°C , and it fell by 16°C during the night before reaching a minimum. What was the minimum temperature?
-

4. A shopper hops in a lift on the 7th floor and travels down 10 levels. Which level does he finish at?
-

5. Your company is in debt to the amount of K2000. You then receive notice that you owe the bank K1500. What is your balance now?
-

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 1

Lesson 4: Multiplying Directed Numbers



You learnt to subtract directed numbers and applied them in real life situations in Lesson 3.



In this lesson, you will:

- multiply directed numbers
- solve problems involving directed numbers

Multiplication of Directed Numbers

Problem: Alicia owes \$6 to each of 4 friends. How much money does she owe?

Solution: The problem above can be solved using integers.

Owing \$6 can be represented by -6. Thus the problem becomes:

$$(-6) \times (+4) \quad \text{or} \quad (-6) (+4)$$

The parentheses () indicate that these integers are being multiplied. In order to solve this problem, we need to know the rules for multiplication of integers.

Rule # 1: The product of a positive integer and a negative integer is a negative integer.

Rule # 2: The product of two negative integers or two positive integers is a positive integer.

We can now use RULE # 1 to solve the problem arithmetically:

$(-6)(+4) = -24$. So Alicia owes K24 . Let's look at some more examples of multiplying integers using these rules.

Example: Find the product of each pair of integers.

Multiplying Integers		
Integers	Product	Rule Used
$(+7) (+3) =$	$+21$	Rule 2
$(+7) (-3) =$	-21	Rule 1
$(-7) (+3) =$	-21	Rule 1
$(-7) (-3) =$	$+21$	Rule 2

Example 2: Find the product of each pair of integers.

Multiplying Integers		
Integers	Product	Rule Used
$(+8)(+4) =$	$+32$ or 32	Rule 2
$(+11)(-2) =$	-22	Rule 1
$(-14)(+3) =$	-42	Rule 1
$(-9)(-5) =$	$+45$ or 45	Rule 2

In each of the above examples, we multiplied two integers by applying the rules at the top of the page. We can multiply three integers, two at a time, applying these same rules. Look at the example below.

Example 3: Find the product of each set of integers.

Multiplying Three Integers		
Integers	Product of First Two Integers and the Third	Product
$(+5)(+3)(+2) =$	$(+15)(+2) =$	30
$(+8)(+2)(-5) =$	$(+16)(-5) =$	-80
$(-6)(+3)(+4) =$	$(-18)(+4) =$	-72
$(-9)(-3)(+2) =$	$(+27)(+2) =$	54
$(-5)(-3)(-5) =$	$(+15)(-5) =$	-75

The Associative Law of Multiplication applies to integers. In Example 3 above, we multiplied the product of the first and second integer by the third integer. We can also solve these problems by multiplying the first integer by the product of the second and third. We will do this in Example 4 below

Example 4: Find the product of each set of integers.

Multiplying Three Integers		
Integers	Product of First Integer and the Last Two	Product
$(+5)(+3)(+2) =$	$(+5)(+6) =$	30
$(+8)(+2)(-5) =$	$(+8)(-10) =$	-80
$(-6)(+3)(+4) =$	$(-6)(+12) =$	-72
$(-9)(-3)(+2) =$	$(-9)(-6) =$	54
$(-5)(-3)(-5) =$	$(-5)(+15) =$	-75

Remember:

Multiplying two integers with like signs yields a positive product, and multiplying two integers with unlike signs yields a negative product. We can multiply three integers, two at a time, applying these same rules.

NOW DO PRACTICE EXERCISE 4

**Practice Exercise 4**

1. Evaluate the following:

- a. $(-11)(30)$
 - b. $(90)(-20)$
 - c. $(3)(45)$
 - d. $(-333)(-3)$
-

2. Give what is asked in the problem:

- a. Multiply (-5) by (-2) . Then add 3 to their product.
 - b. Multiply 3 by (-4) . Then add (-1) to their product.
 - c. Add 5 to (-2) . Then multiply this sum by (-6) .
 - d. Add 2 to (-7) . Then multiply this sum by (-4) .
 - e. From 6 subtract (-2) . Then multiply their difference by 5.
 - f. From (-5) subtract (-3) . Then multiply their difference by (-6) .
-

3. Find the value of :

- (a) $(-5)^2$ b. $(-2)^3$ c. $(-1)^5$ d. $(-4)^3$
-

4. Find the product of :

- a. $(+8)(-2)(+1)$
 - b. $(-4)(-3)(-5)$
 - c. $(-7)(2)(-3)$
 - d. $(6)(3)(-2)$
-

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 2

Lesson 5: Dividing Directed Numbers



You learnt to multiply directed numbers in Lesson 4.

In this lesson, you will:

- divide directed numbers
- solve problems involving directed numbers

Division of Directed Numbers

Problem: Mrs. Jenson owes \$8000 on her car loan. Each of her 4 children is willing to pay an equal share of this loan. Using integers, determine how much money each of her children will pay.



Solution: Owing K8000 can be represented by -8,000. We must divide -8000 by 4 in order to solve this problem. However, we need rules for dividing integers in order to continue.

Rule # 1: The quotient of a positive integer and a negative integer is a negative integer.

Rule # 2: The quotient of two negative integers or two positive integers is a positive integer.

We can now use Rule 1 to solve the problem above arithmetically:

$-8,000 \div +4 = -2,000$. Each of Mrs. Jenson's four children will pay K2,000.

Let's look at some more examples of dividing integers using the above rules.

Example 1: Find the quotient of each pair of integers.

Dividing Integers		
Integers	Quotient	Rule Used
$+24 \div (+12) =$	+2	Rule 2
$+24 \div (-12) =$	-2	Rule 1
$-24 \div (+12) =$	-2	Rule 1
$-24 \div (-12) =$	+2	Rule 2

Example 2: Find the quotient of each pair of integers.

Dividing Integers		
Integers	Quotient	Rule Used
$+27 \div (+3) =$	+9	Rule 2
$+27 \div (-3) =$	-9	Rule 1
$-27 \div (+3) =$	-9	Rule 1
$-27 \div (-3) =$	+9	Rule 2

Example 3: Find the quotient of each pair of integers.

Dividing Integers		
Integers	Quotient	Rule Used
$+99 \div (+11) =$	$+9$	Rule 2
$+80 \div (-16) =$	-5	Rule 1
$-72 \div (+12) =$	-6	Rule 1
$-91 \div (-13) =$	$+7$	Rule 2

Note that multiplication is the inverse operation of division. So since we know how to multiply directed numbers, it is easy to learn how to divide them.

Remember:

- The quotient of a positive integer and a negative integer is a negative integer.
- The quotient of two negative integers or two positive integers is a positive integer.

NOW DO PRACTICE EXERCISE 5

**Practice Exercise 5**

1. Calculate:

- a) $-12 \div (-3)$
 - b) $+90 \div (-3)$
 - c) $+88 \div (+4)$
 - d) $-20 \div (-4)$
-

2. Give what is asked:

- a) Divide (-9) by (-3) . Then multiply the quotient by (-4) .
 - b) Multiply (-9) by (-2) . Then divide the product by (-3) .
 - c) Find the sum of (-6) and 21 . Divide the sum by (-5) .
-

3. A group of 6 people lost K240 trying to win a lottery. What was each person's share of the loss?

4. The temperature falls 10°C in 5 hours one night in Sydney. What is the average temperature change each hour?

5. A large department store in Brisbane has 6 floors below ground level and 9 floors above ground. A tourist on the top level decides to take a lift to the bottom level, stopping every 3 floors to look around briefly. How many stops will the tourist make?

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 1

Lesson 6: Solving Mixed Problems



You learnt to perform various operations on directed numbers .



In this lesson, you will:

add, subtract, multiply and divide directed numbers in mixed problems

The rules for order of operations also apply to directed numbers.

The rules are:

- 1. Perform calculations in brackets from inside out.**
- 2. Next, perform multiplications and divisions in order from left to right.**
- 3. Finally, perform additions and subtractions from left to right**

Example 1

Evaluate: $5 - 6 - 7 + 2$

Solution:

Evaluate: $5 - 6 - 7 + 2$

Write out the question	$5 - 6 - 7 + 2$
Since there are only addition and subtraction involved, evaluate from left to right.	$= -1 - 7 + 2$ $= -8 + 2$ $= -6$

Example 2:

Evaluate: $(-7) \times (4) \div (-2) \times (-2)$

Solution:

Write out the question	$(-7) \times (4) \div (-2) \times (-2)$
Since there are only multiplication and division involved, work from left to right.	$= (-28) \div (-2) \times (-2)$ $= 14 \times (-2)$ $= -28$

Example 3:

Evaluate: $-3 + (-2 \times -6) - 4$

Solution:

Write out the question	$-3 + (-2 \times -6) - 4$
Evaluate brackets first	$= -3 + 12 - 4$
Perform additions and subtractions from left to right	$= 9 - 4$ $= 5$

Example 4:

Evaluate: $-60 \div 5 + 3 \times -4 - 8$

Solution:

Write out the question	$-60 \div 5 + 3 \times -4 - 8$
Divide and multiply first, from left to right	$-12 + -12 - 8$
Perform additions and subtractions from left to right	$= -24 - 8$ $= -32$

Here are some practical problems applying directed numbers.

Example 5

An earth mover is working at a mine. It goes forward 100 metres, then backward 200 metres and, finally, forward again for 75 metres.

- How far is the earth mover from its starting point now.
- The earth mover then goes backward for 55 metres, how far from its starting position does it come to rest?

Solution:

Forward means +100 ; backward means -200 ; another forward means +75

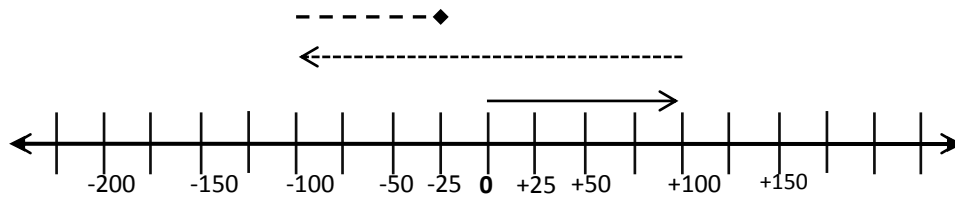
- To find how far the earth mover is from its starting point we can represent it this way: $(+100) + (-200) + (+75) = (-100) + (+75) = -25$

So what does the negative 25 mean ?

Yes you're right ! That the earth mover is **25 metres from the LEFT** of the starting point.

Note: we are talking about distance , so there is no negative distance but rather the negative sign denotes the direction of the object from the starting point.

By using the number line:



Example 6

Bank deposits are listed as positive numbers and withdrawals as negative numbers. Give the final balance for the following account:

+K150, -K17.64, -K36.39, +K48.60, -K72.38, -K26.78, +K16.49, -K44.80

Solution:

$$(+150) + (-17.64) + (-36.39) + (48.60) + (-72.38) + (-26.78) + (+16.49) + (-44.80)$$

Combine all positive numbers together and negative numbers together .

$$(+150) + (+48.60) + (+16.49) = +215.09$$

$$(-17.64) + (-36.39) + (-72.38) + (-26.78) + (-44.80) = -197.99$$

Adding the two answers above : $+215.09 + (-197.99)$. The sum is **17.10**.

What does this mean?

That the withdrawal is less than the deposit that is why the answer is positive.

NOW DO PRACTICE EXERCISE 6

**Practice Exercise 6**

1. Evaluate:

- a. $4 + (6 \times -3) - 2$
 - b. $(-3 \times -5) \times -8$
 - c. $7 + (14 \div -2) - 3$
 - d. $(-6 \times 5) + (-24 \div -6)$
 - e. $(-8 + 2) \times (4 - 10)$
 - f. $(-5 \times -9) - (7 - 10)$
 - g. $(-48 \div -4) \div (-1 + 3)$
-

2. The minimum overnight temperatures for one week at Mt. Wilhelm were 2°C , 5°C , 3°C , 5°C , 3°C , 2°C , 1°C . What was the average temperature for the week?

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 2

TOPIC 1: SUMMARY



This summarizes some of the important ideas and concepts to remember.

Directed numbers are the set of whole numbers and their opposites. This set of numbers are also called the **Set of Integers**. Whole numbers greater than zero are called **positive integers**. Whole numbers less than zero are called **negative integers**. The integer zero is neither positive nor negative, and has no sign. Two integers are opposites if they are each the same distance away from zero, but on opposite sides of the number line. Positive integers can be written with or without a sign.

In previous lessons, you learned how to add, subtract, multiply and divide integers.

There are many rules to remember so it's easy to get confused. This summary will review all the rules learned for operations with integers. You will then be given a mixed set of exercises to complete.

Rules in adding directed numbers

- The sum of two positive integers is a positive integer.
- The sum of two negative integers is a negative integer.
- To find the sum of a positive and a negative integer, find the absolute value of each integer and then subtract these values. The result takes the sign of the integer with the larger absolute value.

Rules in Subtracting directed numbers

- To subtract an integer, add its opposite.

Rules in Multiplying and Dividing directed numbers

- Like signs yield a positive result.
- Unlike signs yield a negative result.

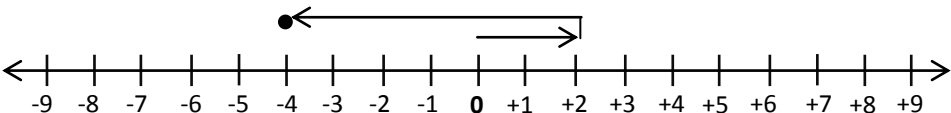
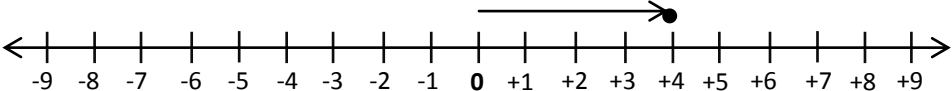
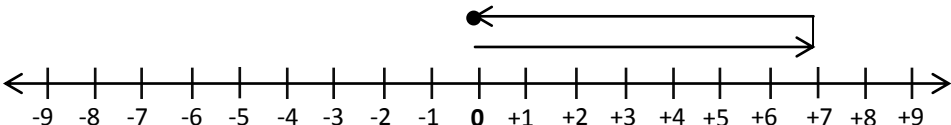
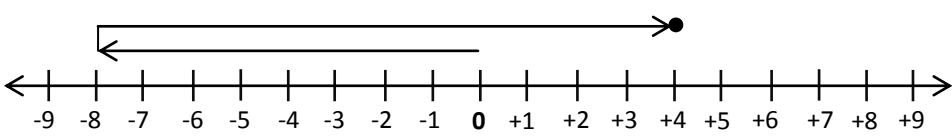
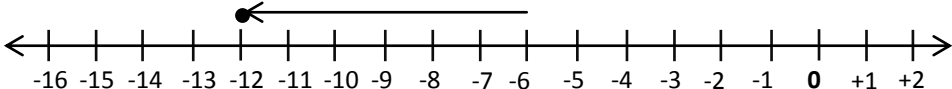
REVISE LESSONS 1- 6. THEN DO TOPIC TEST 1 IN ASSIGNMENT BOOK 2.

ANSWERS TO PRACTICE EXERCISES 1 – 6

Practice Exercise 1

1. a) +3 b) -5 c) -250
d) -4 e) +28
 2. a) If north is a positive direction, south is a negative direction.
b) If right is a positive directions, left is a negative direction
 3. a) Positive 7 b) Up 100 steps
c) North 20km d) 50m above ground
e) 3 days late
 4. a) $-9 < -4$ b) $0 > -10$ c) $7 > -48$
d) $13 > -13$ e) $-15 < 1$
 5. a) Tuesday b) Friday
 6. a) 11, 9, 14, 10 and 20
 7. a) -23, -9, -7, +3, +6 b) +11, +8, 0, -13, -15, -19
-

Practice Exercise 2

1. a) 
- b) 
- c) 
- d) 
- e) 

2. a) +27 b) +6 c) -86
 d) +8 e) +36 f) +15
3. $-12^{\circ}\text{C} + 20^{\circ}\text{C} = +8^{\circ}\text{C}$
4. $-3 + 14 = 11$; So the lift ends up on the 11th floor
5. $12\text{m} - 17\text{m} = -5\text{m}$; So the pool is 5m deep
6. $\$5 - \$12 + \$20 = -\$7 + \$20 = \13 ; So at the end, you have \$13
-

Practice Exercise 3

1.

Subtract	Add the opposite	Result
a) $+8 - -10$	$+8 + -10$	-2
b) $+4 - -5$	$+4 + -5$	-1
c) $-2 - +6$	$-2 + +6$	$+4$
d) $+10 - -7$	$+10 + -7$	$+3$

2. a) -8 b) -40 c) +30
 d) -153 e) -18
3. $12^{\circ}\text{C} - 16^{\circ}\text{C} = -4^{\circ}\text{C}$; so the minimum temperature is -4°C
4. $7 - 10 = -3$; so he finishes on the 3rd floor underground
5. $-K2000 - K1500 = -K3500$
-

Practice Exercise 4

1. a) -330 b) -1800 c) +135 d) +999
2. a) $-5 \times -2 + 3 = +10 + 3 = +13$
 b) $3 \times -4 + -1 = -12 + -1 = -13$
 c) $(-2 + 5) \times -6 = 3 \times -6 = -18$
 d) $(-7 + 2) \times -4 = -5 \times -4 = +20$
 e) $(6 - -2) \times 5 = 8 \times 5 = 40$
 f) $(-5 - -3) \times -6 = -2 \times -6 = 12$
3. a) $(-5)^2 = -5 \times -5 = +25$ b) $(-2)^3 = -2 \times -2 \times -2 = -8$
 c) $(-1)^5 = -1 \times -1 \times -1 \times -1 \times -1 = -1$ d) $(-4)^3 = -4 \times -4 \times -4 = -64$.
4. a) -16 b) -60 c) +42 d) -36
-

Practice exercise 5

1. a) +4 b) -30 c) +22 d) +5

2. a) $-9 \div -3 \times -4 = +3 \times -4 = -12$
b) $-9 \times -2 \div -3 = 18 \div -3 = -6$
c) $(-6 + 21) \div -5 = 15 \div -5 = -3$
3. $K240 \div 6 = K40$
4. $10^{\circ}\text{C} \div 5 = 2^{\circ}\text{C}$
5. Total no. of floors = 15 floors.
If tourist stops every 3 floors; $15 \div 3 = 5$,
Therefore the tourist will make 5 stops.
-

Practice Exercise 6

1. a) -16 b) -120 c) -3 d) -26
e) +36 f) +48 g) 6
2. $(2^{\circ}\text{C} + 5^{\circ}\text{C} + 3^{\circ}\text{C} + 5^{\circ}\text{C} + 3^{\circ}\text{C} + 2^{\circ}\text{C} + 1^{\circ}\text{C}) \div 7 = 21^{\circ}\text{C} \div 7 = 3^{\circ}\text{C}$
-

END OF TOPIC 1

TOPIC 2

INDICES

Lesson 7:	Squares and Square Roots
Lesson 8:	Multiplication Laws of Indices
Lesson 9:	Division laws of Indices
Lesson 10:	Zero and Negative Indices
Lesson 11:	Scientific Notation
Lesson 12:	Combined Index Operations

TOPIC 2: INDICES

Introduction



Powers or indices provide a convenient notation when we need to multiply a number by itself several times.

In this topic we will explain how indices are written and state the Laws that are used for manipulating them.

This topic deals with indices and scientific notation. After completing these lessons, the student should have a clear understanding of the following:

- Changing numbers to index form and vice versa
- Using the terms base, power, index and exponents
- Using the index laws to simplify expressions
- Defining and using zero and negative indices and
- Expressing very large and very small numbers in scientific notation and vice versa.

Lesson 7: Squares and Square Roots



In the past lessons in Grade 8, you learnt about indices. You have learned that indices or powers are an important part of mathematics as they are numbers that indicate that a number is multiplied by itself for a given number of times.



In this lesson, you will:

- define squares of numbers
- find squares of numbers
- define square roots
- find square roots
- find the square and square root using the table of squares and square root.

The Square of a Number

To be able to understand this lesson easily we will first discuss about squares then square roots.

Let us recall some definition of terms in your Grade 8 lessons on indices.

A repeated multiplication is written as a **power**.

The number that is multiplied is the **base** and is written in normal font size.

The number of times the base is multiplied is given by the **index** and is written as a small number at the top right of the base. The index is also called the **exponent**.

Extended form		Power or Index Form		Numerical Value
4×4	=	4^2	=	16

A number raised to the power 2 is called a **square**.

From the above example, **16** is the square of 4.

How to Square a Number

To square a number, just multiply the number by itself.

Example:

- What is 3 squared?
- What is 10 squared?
- What is 35 squared ?

Solution:

$$\begin{array}{llllll} \text{a) } 3 \text{ squared} & = & 3^2 & = & 3 \times 3 & = & \mathbf{9} \\ \text{b) } 10 \text{ squared} & = & 10^2 & = & 10 \times 10 & = & \mathbf{100} \\ \text{c) } 35 \text{ squared} & = & 35^2 & = & 35 \times 35 & = & \mathbf{1225} \end{array}$$

“the exponent 2 says the number must be multiplied by itself **twice**”

Squares from 1^2 to 6^2

$$\begin{array}{llll} 1 \text{ Squared} & = & 1^2 & = 1 \times 1 = \mathbf{1} \\ 2 \text{ Squared} & = & 2^2 & = 2 \times 2 = \mathbf{4} \\ 3 \text{ Squared} & = & 3^2 & = 3 \times 3 = \mathbf{9} \\ 4 \text{ Squared} & = & 4^2 & = 4 \times 4 = \mathbf{16} \\ 5 \text{ Squared} & = & 5^2 & = 5 \times 5 = \mathbf{25} \\ 6 \text{ Squared} & = & 6^2 & = 6 \times 6 = \mathbf{36} \end{array}$$

You can also find the squares of a number on the multiplication table.

1	2	3	4	5	6
2	4	6	8	10	12
3	6	9	12	15	18
4	8	12	16	20	24
5	10	15	20	25	30
6	12	18	24	30	36

Negative Numbers

You can also square **negative numbers**.

Example: What happens when you square (-5)?

Answer: $(-5) \times (-5) = \mathbf{25}$ (because a negative times a negative gives a positive)

The square of a negative number is a positive number.

That was interesting!

Just the same as if you had squared a positive number:

$$\begin{array}{rcl} 5 \times 5 & = & 25 \\ -5 \times -5 & = & 25 \end{array} \quad \leftarrow \text{same answer}$$

Note: if someone says "**negative 5 squared**" do you:

- Square the 5, then do the negative?
- Or do you square (-5)?

You get different answers:

$$\begin{array}{ll} \text{Square 5, then do the minus} & \text{Square } (-5) \\ -(5 \times 5) = -25 & (-5) \times (-5) = +25 \end{array}$$

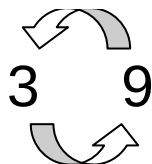
Always make it clear what you mean, and that is what the "()" are for.

Square Roots

We have opposite for addition and multiplication. In the same way, the opposite of finding the square (known as squaring) is to find the **square root**.

A **square root** is the reverse operation of squaring.

Square Root



Square

3 squared is 9, so a **square root of 9 is 3**

A square root of **9** is **3**, because **when 3 is multiplied by itself** you get **9**.

The square root of a number is a number which when *multiplied by itself* gives the original number.

Here are more squares and square roots:

SQUARE		
4		16
5		25
6		36
SQUARE ROOT		

This table means that: 4 square is **16**. And the square root of 16 is **4**.

- When you square a number the result becomes bigger.
- When you get the square root of a number the result becomes smaller.

The Square Root Symbol



This is the special symbol that means "square root", it is sort of like a tick, and actually started hundreds of years ago as a dot with a flick upwards.

It is called the **radical or the surd sign** and always makes math look important!

You can use it like this: $\sqrt{9} = 3$

which reads "**square root of 9 equals 3**"

Example: What is $\sqrt{25}$?

Well, we just happen to know that $25 = 5 \times 5$, so if you multiply 5 by itself (5×5) you will get 25.

So the answer is: $\sqrt{25} = 5$

Example: What is $\sqrt{36}$?

Answer: $6 \times 6 = 36$, so $\sqrt{36} = 6$

Perfect Squares

Below is the table of perfect squares. Perfect squares are the squares of the whole numbers:

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	etc
Perfect Squares:	1	4	9	16	25	36	49	64	81	100	121	144	169	196	225	...

Try to remember at least the first 10 of those.

Example: Use this table to estimate $\sqrt{55}$.

Solution: 55 is between 49 and 64.
So, $\sqrt{55}$ is between 7 and 8.

Example: Estimate the $\sqrt{110}$

Solution: 110 is between 100 and 121.
So, $\sqrt{110}$ is between 10 and 11.

Squaring a decimal or a fraction

When we square a decimal or a fraction we use brackets.

Example 1

Find the square of 0.25

Solution: $(0.25)^2 = (0.25)(0.25) = 0.0625$

Example 2

Find the square of $\frac{3}{4}$.

Solution: $\left(\frac{3}{4}\right)^2 = \left(\frac{3}{4}\right)\left(\frac{3}{4}\right) = \frac{9}{16}$

NOW DO PRACTICE EXERCISE 7

**Practice Exercise 7**

1. Find the squares of these numbers.

- a) 17
 - b) -15
 - c) 0.7
 - d) 0.35
 - e) $\frac{2}{3}$
-

2. Using the table of perfect squares, find the square root of the following:

- a) $\sqrt{64}$
 - b) $\sqrt{81}$
 - c) $\sqrt{16}$
 - d) $\sqrt{169}$
-

3. Evaluate:

- a) $(-9)^2 + 2^2$
 - b) $(21)^2 - (-7)^2$
-

4. Between which two consecutive whole numbers is each of the following square roots?

- a) $\sqrt{14}$
 - b) $\sqrt{20}$
 - c) $\sqrt{6}$
-

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 2

Lesson 8: Multiplication Laws of Indices



You were introduced to the product law in Gr 8 Mathematics. You have learned that when multiplying expressions with same bases, the indices are added.

In symbols $a^m \times a^n = a^{m+n}$ where **a** is the base and **n** and **m** are the indices.



In this lesson, you will:

- define and identify index expressions
- identify multiplication laws of indices
- multiply index expressions

Index Expressions

We know that the expression $2 \times 2 \times 2 \times 2 \dots$ is called the **expanded form**.

The base which is number 2 is being multiplied by itself 4 times.

This expanded form can be written into a form called **index or exponential form**.

Expanded Form	Index Form
$2 \times 2 \times 2 \times 2$	2^4

We read this as “2 raised to the power of 4” or the 4th power of 2.

In the expression 2^4 , “2” is called the base and “4” is called the index or exponent. It is often more convenient to use the index form than the expanded form.

Example 1

Expanded form

Index form

$$3 \times 3 \times 3$$

$$3^3$$

$$1 \times 1 \times 2 \times 2 \times 2$$

$$1^2 \times 2^3$$

$$5 \times 5 \times 5 \times 7 \times 7 \times 9$$

$$5^3 \times 7^2 \times 9$$

Repeated pronumeral factors may also be written using index notation.

Example 2 Simplify each of the following using index notations.

a) $m \cdot m \cdot m$

b) $y \cdot y \cdot y \cdot y \cdot z \cdot z$

c) $9 \cdot a \cdot a \cdot b \cdot c \cdot c$

Solution:

a)	$m \cdot m \cdot m$	m^3
b)	$y \cdot y \cdot y \cdot y \cdot z \cdot z$	$y^4 z^2$
c)	$9 \cdot a \cdot a \cdot b \cdot c \cdot c$	$9a^2 bc^2$

Note:

- The dot means multiplication so as not to confuse you with the pronumeral x.
- Writing the numbers and pronumerals side by side means that the operation used is multiplication.

Example 4

Show by writing the expanded form of the following:

a) $a^4 b^3 c^2$	b) $2^3 x^2 z^5$
------------------	------------------

Solution

$$a) a^4 b^3 c^2 = a \cdot a \cdot a \cdot a \cdot b \cdot b \cdot b \cdot c \cdot c$$

$$b) 2^3 x^2 z^5 = 2 \cdot 2 \cdot 2 \cdot x \cdot x \cdot z \cdot z \cdot z \cdot z \cdot z$$

Now that you know how to express expanded form into index form and vice versa, we are ready to use the multiplication laws of indices to simplify index expressions.

Multiplication Laws of indices

Rule #1 : SAME BASE

When multiplying numbers with the same base, we **ADD** the indices.

In symbols:

$$a^m \times a^n = a^{m+n}$$

Rule #2 : POWERS OF POWERS

When raising a power to another power, we **MULTIPLY** the indices.

In symbols:

$$(a^m)^n = a^{m \times n}$$

Rule #3: IDENTITY LAW

When a number is multiplied by 1, the answer is the number **ITSELF**.

In symbols:

$$a^m \times 1 = a^m$$

Example 3

Expand and simplify $(2^3)(2^5)$

STEPS:

Solutions:

- | | |
|---|---|
| 1. Write down the expression | $2^3 \cdot 2^5$ |
| 2. Expand according to the indices | $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$ |
| 3. Write as a single power
(by counting the number of factors) | 2^8 |

Example 4

Expand and simplify $(y^2)(y^3)$

- | | |
|---|-------------------------------------|
| 1. Write down the expression | $(y^2)(y^3)$ |
| 2. Expand according to the indices | $y \cdot y \cdot y \cdot y \cdot y$ |
| 3. Write as a single power
(by counting the number of factors) | y^5 |

By using the multiplication index law 1:

- $(2^3)(2^5) = 2^{3+5} = 2^8$
- $(y^2)(y^3) = y^{2+3} = y^5$

Example 5:

Expand and simplify:

a) $(3^2)^3$

b) $(b^4)^5$

c) $(m^2)^4$

Solutions:

- | | | |
|----|------------------------------|---|
| a) | 1. Write down the expression | $(3^2)^3$ |
| | 2. Expand the powers | $3^2 \cdot 3^2 \cdot 3^2$ |
| | 3. Write as a single power | $3^{2+2+2} = 3^6$ |
| b) | 1. Write down the expression | $(b^4)^5$ |
| | 2. Expand the powers | $b^4 \cdot b^4 \cdot b^4 \cdot b^4 \cdot b^4$ |
| | 3. Write as a single power | $b^{4+4+4+4+4} = b^{20}$ |
| c) | 1. Write down the expression | $(m^2)^4$ |
| | 2. Expand the powers | $m^2 \cdot m^2 \cdot m^2 \cdot m^2$ |
| | 3. Write as a single power | $m^{2+2+2+2} = m^8$ |

By using the multiplication index law 2:

a) $(3^2)^3 = 3^{2 \times 3} = 3^6$

b) $(b^4)^5 = b^{4 \times 5} = b^{20}$

c) $(m^2)^4 = m^{2 \times 4} = m^8$

Example 6

Simplify the following:

a) $2^5 \times 1$

b. $d^4 \times 1$

c. $m^{10} \times 1$

By using multiplication index law 3:

b) $2^5 \times 1 = 2^5$

b. $d^4 \times 1 = d^4$

c. $m^{10} \times 1 = m^{10}$

NOW DO PRACTICE EXERCISE 8

**Practice Exercise 8**

1. Express the following in index form:

a) $m \cdot m \cdot m \cdot m \cdot m$

b) $a \cdot a \cdot a \cdot a \cdot a \cdot a \cdot a$

2. Use the index laws to simplify:

a) $M^3 \cdot M^5$

b) $t^{10} \cdot t^9$

c) $v \cdot v^5 \cdot v^7$

3. Use the index laws to simplify:

a) $(5^2)^4$

b) $(17^6)^3$

c) $(k^8)^2$

d) $(z^{10})^6$

4. Use the index laws to simplify:

a) $(6^3)^3$

b) $5^5 \cdot 5^9$

c) $a^2 \cdot a \cdot a^5$

d) $1 \cdot m$

e) $(4^2)^1$

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 2

Lesson 9: Division Laws of Indices



You have learned how to apply multiplication laws of indices in the last lesson.



In this lesson, you will:

- identify division laws of indices
- divide index expressions.

In multiplication of same bases, we **ADD** the powers.

Now, let us consider the following expressions which involve division of indices:

By expansion we have
$$\frac{a^4}{a^2} = \frac{\cancel{a} \cdot \cancel{a} \cdot a \cdot a}{\cancel{a} \cdot \cancel{a}} = a^2$$

You will notice that, $4 - 2 = 2$.

i.e. subtracting indices shows that: $a^4 \div a^2 = a^{4-2} = a^2$.

Note that $4 - 2 = 2$ (subtracting indices). Thus, we can safely deduce that when index numbers having the same base are divided, the result is obtained as the base raised to the difference of their indices.

This is our Division Rule #1 written as:

$$\frac{a^m}{a^n} = a^{m-n}$$

(SAME BASE)

Example 1 Expand and simplify $\frac{2^7}{2^5}$.

Solutions:

1. Write down the expression

$$\frac{2^7}{2^5}$$

2. Expand the powers

$$\frac{\cancel{2} \cdot \cancel{2} \cdot 2 \cdot \cancel{2} \cdot \cancel{2} \cdot 2 \cdot 2}{\cancel{2} \cdot \cancel{2} \cdot 2 \cdot \cancel{2} \cdot \cancel{2}}$$

3. Cancel as many factors as possible
Write as a single power

$$2^2$$

Using the division rule: $2^7 \div 2^5 = 2^{7-5} = 2^2$

Example 2 Expand and simplify $\frac{4x^3}{2x^2}$

1. Write down the expression

$$\frac{4x^3}{2x^2}$$

2. Expand the powers

$$\frac{4 \bullet x \bullet x \bullet x}{2 \bullet x \bullet x}$$

3. Cancel as many factors as possible **2x**
Divide the numbers. Write as a single power

Using the division rule: $\frac{4x^3}{2x^2} = 2x^{3-2} = 2x$

Now observe the following examples below:

Example 3 Expand and simplify $\frac{x^3}{x^3}$.

Solution

1. Write down the expression

$$\frac{x^3}{x^3}$$

2. Expand the powers

$$\frac{x \bullet x \bullet x}{x \bullet x \bullet x}$$

3. Cancel as many factors as possible **1 • 1 • 1**
Write as a single power **1**

What happened after you cancelled as many factors in the problem?
Yes, you're right. All factors cancelled out and the answer is 1.

Thus, we can safely deduce that when index numbers having the same base and powers are divided, the result obtained from the difference of their indices is zero.

This is our Division Rule #2 written as:

$$\frac{a^m}{a^m} = a^{m-m} = a^0 = 1$$

(SAME BASE AND POWER)

And because we are dividing an expression by itself which is equal to 1, therefore any numeral, pronumeral or expression raised to zero is equal to one.

Any number raised to zero is equal to 1.

Example 1 Expand and simplify $\frac{15x^7}{3x^7}$.

Solution

1. Write down the expression

$$\frac{15x^7}{3x^7}$$

2. Expand the powers

$$\frac{15 \bullet x \bullet x \bullet x \bullet x \bullet x \bullet x \bullet x}{3 \bullet x \bullet x \bullet x \bullet x \bullet x \bullet x \bullet x}$$

3. Cancel as many factors as possible

$$\frac{15}{3}$$

Write as a single power

5

By using the division index law 2: $\frac{15x^7}{3x^7} = \frac{15}{3}x^{7-7} = 5x^0 = 5(1) = 5$

Example 2 Simplify and expand $\frac{18x^3}{6x^3}$

By using the division index law 2: $\frac{18x^3}{6x^3} = \frac{18}{6}x^{3-3} = 3x^0 = 3(1) = 3$

NOW DO PRACTICE EXERCISE 9

**Practice Exercise 9**

1. Simplify and give the answer in index form:

a) $7^9 \div 7^4$

b) $3^4 \div 3^3$

c) $8^4 \div 8$

2. Simplify:

a) $h^9 \div h^4$

b) $16f^{10} \div 4f^9$

c) $27y^8 \div 9y$

d) $A^{20} \div A^{20}$

3. Simplify and express answer in index form:

a) $\frac{12^7}{12^3}$

b) $\frac{4^6}{4^4}$

c) $\frac{e^6}{e^3}$

d) $\frac{h^9}{h}$

e) $\frac{15^3}{15^2}$

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 2

Lesson 10: Zero and Negative Indices



In the previous lesson, you learnt two rules in division of indices.

In this lesson you will:

- define zero and negative indices
- simplify expressions with zero indices
- simplify negative indices.

This lesson is an extension of the previous lesson where you will encounter zero and negative indices.

The Zero Index

When simplifying indices on division of similar bases where subtraction of indices happens, there is a tendency that the index becomes zero.

When does this happen and how do we simplify a zero index further?

Here is an example.

Simplify: $b^5 \div b^5$

In this case is $b^{5-5} = b^0$.

Also it can be solved as b^5 divided by $b^5 = 1$.

This shows that $b^0 = 1$.

From the above example, zero index happens when the indices are the same.

Now try to find answers to the following:

a) 7^0

b) 90^0

c) a^0

All the answers will be 1. If you got the right answers then BRAVO you have made great progress.

The Zero Index

$$\frac{a^m}{a^m} = a^{m-m} = a^0 = 1$$

Any number, pronumeral or expression raised to zero is equal to **ONE**.

Example: Simplify the zero indices

a) $3y^0$

c) $a^0 + b^0$

e) $7m^0 + 2$

b) $(5m)^0$

d) $(x^3)^0$

Solution:

a) $3(1) = 3$

c) $1 + 1 = 2$

e) $7 + 2 = 9$

b) 1

d) 1

The Negative Index

All the indices so far that we have seen have been positive or zero.

If we had, $2^3 \div 2^5$, the answer, according to the second index law would be, 2^{3-5} .

$$\text{i.e. } 2^3 \div 2^5 = 2^{3-5} = 2^{-2}$$

But this could also be written in this way:

$$\frac{2^3}{2^5} = \frac{2 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2} = \frac{1}{2 \cdot 2} = \frac{1}{2^2}$$

$$\text{So, } 2^{-2} = \frac{1}{2^2}$$

In general, the meaning of a negative index can be summarized by the rules:

The Negative Index

$$\begin{aligned} a^m \div a^n &= a^{m-n} \quad (n > m) \\ &= a^{-m} \\ &= \frac{1}{a^m} \end{aligned}$$

Any number or pronumeral raised to a negative index is equal to its **RECIPROCAL**.

Example 1

Write down the meaning of:

a) k^{-19}

Solution:

$$k^{-19} = \frac{1}{k^{19}}$$

b) m^{-15}

$$m^{-15} = \frac{1}{m^{15}}$$

Example 2

Write with a negative index:

a) $\frac{1}{a^5}$

$$\frac{1}{a^5} = a^{-5}$$

b) $\frac{1}{y^7}$

$$\frac{1}{y^7} = y^{-7}$$

Example 3

Write down the meaning of

Solution:

a) $3m^{-2}$

$$3m^{-2} = (3)(m^{-2}) = (3)\left(\frac{1}{m^2}\right) = \frac{3}{m^2}$$

b) $(3m)^{-2}$

$$(3m)^{-2} = \frac{1}{(3m)^2} = \frac{1}{9m^2}$$

Example 4

a) Use the index laws to simplify: $\frac{a^4}{a^5}$

b) Expand and simplify: $\frac{a^4}{a^5}$

c) Hence show that $a^{-1} = \frac{1}{a}$

Solution:

a. Use the index laws to simplify: $\frac{a^4}{a^5}$

$$\frac{a^4}{a^5} = a^{4-5} = a^{-1}$$

b. Expand and simplify $\frac{a^4}{a^5}$

$$\frac{a^4}{a^5} = \frac{a \cdot a \cdot a \cdot a}{a \cdot a \cdot a \cdot a \cdot a} = \frac{1}{a}$$

c. Hence show that $a^{-1} = \frac{1}{a}$

$$\text{Since } \frac{a^4}{a^5} = a^{-1} \text{ and } \frac{a^4}{a^5} = \frac{1}{a} \text{ then } a^{-1} = \frac{1}{a}$$

NOW DO PRACTICE EXERCISE 10

**Practice Exercise 10**

1. Evaluate:

a) y^0 b) $(6z)^0$ c) $3m^0$ d) $a^0 + b^0$

2. Write down the meaning of :

a) t^{-10} b) $3k^{-1}$ c) $(2y)^{-5}$ d) $3t^{-4}$

3. Expand and simplify :

a. $\frac{x^3}{x^3}$

b) $\frac{22y^5}{11y^5}$

c) $\frac{m^5}{m^9}$

d) $\frac{x}{x^{10}}$

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 2

Lesson 11: Scientific Notation



You have learned about the different laws of indices in the previous lessons.



In this lesson you will:

- define scientific notation
- write number in scientific notation
- write number in scientific notation as ordinary number
- solves problems involving scientific notation

The distance of Mars from the sun is approximately 229 000 000 kilometres.

The diameter of the hydrogen atom is 0.000 000 025 4 metres.

These statements represent very large and very small numbers respectively.

How can these numbers be written in a more convenient way?

Scientists have invented a more convenient method of writing very large and very small numbers like the ones above. It is called **scientific notation or standard notation**.

Scientific notation is a way of writing numbers that are too big or too small in a convenient decimal form.

To write a number in scientific notation, it is written as the product of a number between 1 and 10 and a power of 10.

In symbols:

$$a \times 10^b$$

This means “**a** times ten raised to a power of **b**”

Where: **b** (the exponent) is an integer

a (the coefficient) is any real number

Example

Standard Decimal Notation	Scientific Notation
2	2×10^0
600	6×10^2
5 432. 876	$5.432\ 876 \times 10^3$
7 630 000 000	7.63×10^9
0.3	3×10^{-1}
0.000 000 006 18	6.18×10^{-9}

Now see the following examples.

Example 1

State whether or not the following numbers are written in scientific notation:

- a) 6.7×10^5 c) 2.96×10^{-7} e) 3.65×1000
 b) 23×10^5 d) 480 000

Solution

- a) YES, as the first number (6.7) is between 1 and 10 and it is multiplied by a power of 10. (10^5)
 b) No, because the first number (23) is not between 1 and 10.
 c) YES, as the first number (2.96) is between 1 and 10 and it is multiplied by a power of 10 (10^{-7})
 d) No, as it is not written as a number between 1 and 10 multiplied by a power of 10.
 e) No because the second number (1000) is not expressed as a power of 10.

Example 2

Write the following numbers in scientific notation.

Large Numbers (Positive Numbers)

Solution:

- a. 5 000 000 $5\,000\,000 = 5 \times 1\,000\,000 = \mathbf{5 \times 10^6}$
 b. 40 000 $40\,000 = 4 \times 10\,000 = \mathbf{4 \times 10^4}$.
 c. 5300 $5300 = 5.3 \times 1000 = \mathbf{5.3 \times 10^3}$
 d. 284 000 $284\,000 = 2.84 \times 100\,000 = \mathbf{2.84 \times 10^5}$

Example 3

Write the following numbers in scientific notation.

Small Numbers (Decimal Numbers)

- (a) 0.004

Solution:

$$0.004 = 4 \times 0.0001 = 4 \times \frac{1}{1000} = 4 \times \frac{1}{10^3} = \mathbf{4 \times 10^{-3}}$$

(b) 0.000 009

$$\begin{aligned}\text{Solution: } 0.000\ 009 &= 9 \times 0.000\ 001 = 9 \times \frac{1}{1\,000\,000} \\ &= 9 \times \frac{1}{10^6} = \mathbf{9 \times 10^{-6}}\end{aligned}$$

The following examples show a quicker way of writing numbers in scientific notation.

Example 1

Write 246 000 in scientific notation

Step 1 Move the decimal point so that it is positioned between the first and second digits of the number. This always produces a number between 1 and 10. In this case we get 2.46000.

Step 2 Count the number of places back to the original position of the decimal point in the number.

$$\begin{aligned}2.46000 \quad \text{number of places} &= 5 \text{ to the right} \\ &= +5 \text{ (this becomes the power of 10)}\end{aligned}$$

$$\text{So, } 246\ 000 = \mathbf{2.46 \times 10^5}$$

Example 2

Write 0.000 71 in scientific notation

Step 1 Move the decimal point so it is positioned between the first and the second non-zero digits of the number. In this case we get 7.1.

Step 2 Count the number of places back to the original position of the decimal point in the number.

$$\begin{aligned}0.000\ 71 \quad \text{number of places} &= 4 \text{ to the left} \\ &= -4 \text{ (this becomes the power of 10)}\end{aligned}$$

$$\text{So, } 0.000\ 71 = \mathbf{7.1 \times 10^{-4}}$$

Remember, however many spaces the decimal point needs to be moved to get to its original position, that's the power of 10. If you have a smaller number, (smaller than 1), then the power of 10 is **negative**. If it is a large number (bigger than 1), then the power of 10 is **positive**.

Note: A negative on an exponent and a negative on a number mean two different things. For example:

$$-0.00054 = -5.4 \times 10^{-4}$$

$$-54000 = -5.4 \times 10^4$$

$$0.00054 = 5.4 \times 10^{-4}$$

$$54\ 000 = 5.4 \times 10^4$$

Actually, converting between “standard regular decimal notation” and scientific notation is even simpler than what has been shown so far because all you really need to do is count decimal places.

Changing Scientific Numbers to Ordinary Numbers

Scientific numbers can also be expressed as whole numbers or decimal numbers.

Example: Write the following numbers as ordinary decimal numerals

With positive index

$$\begin{aligned} \text{(a)} \quad 6 \times 10^5 \quad \text{Solution:} \quad 6 \times 10^5 &= 6 \times 100\,000 \\ &= 600\,000 \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad 3.94 \times 10^6 \quad \text{Solution:} \quad 3.94 \times 10^6 &= 3.94 \times 1\,000\,000 \\ &= 3\,940\,000 \end{aligned}$$

Example Write as an ordinary decimal numeral

With negative index

$$\begin{aligned} \text{(a)} \quad 5 \times 10^{-2} \quad \text{Solution:} \quad 5 \times 10^{-2} &= 5 \times \frac{1}{100} \\ &= 5 \times \frac{1}{10^2} \\ &= 5 \times 0.01 \\ &= 0.05 \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad 7 \times 10^{-6} \quad \text{Solution:} \quad 7 \times 10^{-6} &= 7 \times \frac{1}{1\,000\,000} \\ &= 7 \times \frac{1}{10^6} \\ &= 7 \times 0.000\,001 = 0.000\,007 \end{aligned}$$

The quicker way of changing scientific numbers to ordinary numbers is by **reversing** the process or steps above.

Example 1 Write 6.48×10^6 as an ordinary number.

Solution Reversing the process -- since the power of 10 is +6, then the decimal point is moved back 6 places to the **right**.

$$\text{i.e.} \quad 6.480000 \quad \text{Hence, } 6.48 \times 10^6 = 6\,480\,000$$

Example 2 Write 3.51×10^{-6} as an ordinary number.

Solution: Reversing the process since the power of 10 is -6, then the decimal point is moved back 6 places to the **left**.

$$\text{i.e.} \quad 0.000\,003.51 \quad \text{Hence, } 3.51 \times 10^{-6} = 0.000\,00351$$

Scientific Notation and Problem Solving

To perform operations in scientific notations, index laws will be applied.

Example Use the index laws to calculate the following, leaving your answers in scientific notation:

(a) $(3 \times 10^{15}) \times (6 \times 10^{-7})$

$$\begin{aligned}\text{Solution: } (3 \times 10^{15}) \times (6 \times 10^{-7}) &= (3 \times 6) \times (10^{15} \times 10^{-7}) \\ &= (1.8 \times 10^1) \times 10^8 \\ &= 1.8 \times (10^1 \times 10^8) \\ &= 1.8 \times 10^9\end{aligned}$$

(b) $(8 \times 10^{-4}) \div (2 \times 10^6)$

$$\begin{aligned}\text{Solution: } (8 \times 10^{-4}) \div (2 \times 10^6) &= (8 \div 2) \times (10^{-4} \div 10^6) \\ &= 4 \times 10^{-4-6} \\ &= 4 \times 10^{-10}\end{aligned}$$

(c) $(5 \times 10^7)^3$

$$\begin{aligned}\text{Solution: } (5 \times 10^7)^3 &= 5^3 \times (10^7)^3 \\ &= 125 \times 10^{21} \\ &= (1.25 \times 10^2) \times 10^{21} \\ &= 1.25 \times (10^2 \times 10^{21}) \\ &= 1.25 \times 10^{23}\end{aligned}$$

Example

A box has a length of 8×10^4 cm, a width of 5×10^4 cm and a height of 1.2×10^4 cm. What is the volume?

Solution: Volume of a box = length $\times$ width $\times$ height

$$V = (8 \times 10^4) (5 \times 10^4) (1.2 \times 10^4) = 48.0 \times 10^{12} = \mathbf{4.8 \times 10^{13}}$$

NOW DO PRACTICE EXERCISE 11

**Practice Exercise 11**

1. Express the following numbers in scientific notation.
 - (a) The number of hairs on a person's head is approximately 129 000.
 - (b) The diameter of a hydrogen atom is 0.000 000 002 54 centimetres.
-

2. Express the following as ordinary numbers.
 - (a) The distance of Mars from the Earth is 7.83×10^7 kilometres
 - (b) A microsecond is equivalent to 2.5×10^{-9} hours.
-

3. Express in scientific notation.
 - (a) 4970
 - (b) 9 310 000
-

4. Express in scientific notation
 - (a) 0.08
 - (b) 0.000 529
 - (c) 0.426
-

5. Write the basic numeral for:
 - (a) 8.74×10^1

(b) 1.075×10^3

(c) 5×10^{-6}

(d) 6.3×10^{-3}

-
6. The average speed of the Earth around the Sun is approximately 10^5 km/h. How many days would it take the Earth to travel 9.6×10^8 km.

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 2

Lesson 12: Combined Index Operations



You have gone through all the laws of indices you need for this topic. In the previous lessons, you have learned when to add, subtract and multiply indices as well as how to simplify expressions including zero and negative indices.



In this lesson you will:

- apply all index laws to solve problems

It is good to recall all the rules on indices that we learnt from the previous lessons before we start solving more complicated problems.

RULE	IN SYMBOLS	EXPLANATION
Multiplication Rule	$a^m \times a^n = a^{m+n}$	When multiplying indices of the same bases, add the powers.
Division Rule	$a^m \div a^n = a^{m-n}$	When dividing indices of the same base, subtract the powers.
Power Rule	$(a^m)^n = a^{mn}$	When raising power to a given power, multiply the powers.
Zero Index Rule	$a^0 = 1$	Any number raised to the power of zero is equal to ONE (1).
Negative Index Rule	$a^{-m} = \frac{1}{a^m}$	Any number raised to a negative index is equal to its RECIPROCAL.

You can make use of these rules you learnt to solve and simplify combined operations with powers.

Example 1 Simplify $(2m^2)^3$

Solution: by Expansion

$$\begin{aligned}
 (2m^2)^3 &= (2m^2)(2m^2)(2m^2) \\
 &= (2 \cdot 2 \cdot 2)(m^2)(m^2)(m^2) \\
 &= 8m^{2+2+2} \\
 &= 8m^6
 \end{aligned}$$

by Index Rule (power rule)

$$\begin{aligned}
 (2m^2)^3 &= 2^{1 \times 3} m^{2 \times 3} \\
 &= 2^3 m^6 \\
 &= 8m^6
 \end{aligned}$$

Example 2 Simplify $(2m^2y^5)^4$

Solution: By Expansion

$$\begin{aligned}(2m^2y^5)^4 &= (2)(2)(2)(2)(m^2)(m^2)(m^2)(m^2)(y^5)(y^5)(y^5)(y^5) \\ &= 2^4 m^{2+2+2+2} y^{5+5+5+5} \\ &= \mathbf{16m^8y^{20}}\end{aligned}$$

By Index Rule

$$\begin{aligned}(2m^2y^5)^4 &= 2^{1 \times 4} m^{2 \times 4} y^{5 \times 4} \\ &= 2^4 m^8 y^{20} \\ &= \mathbf{16 m^8 y^{20}}\end{aligned}$$

From the two examples above, you will notice that the power law was used not only for a single base but also to a **product of bases** such as numbers and pronumerals and products of pronumerals.

Note also that power law only applies to products of bases. What do we mean by these?

$$\begin{aligned}(a^2b^3)^2 &= a^{2 \times 2} b^{3 \times 2} \\ &= a^4b^6\end{aligned}$$

Product of bases therefore power rule can be applied

$$\left. \begin{aligned}(a^2 + b^3)^2 \\ (a^2 - b^3)^2 \\ (a^2 \div b^3)^2\end{aligned} \right\} \rightarrow \text{These are NOT products of bases so the power law cannot be applied.}$$

Example 3 Simplify $(m^2n^5)(mn^6)$

Solution:

$$\begin{aligned}(m^2n^5)(mn^6) &= (m^2)(m)(n^5)(n^6) \\ &= m^{2+1} \times n^{5+6} \\ &= \mathbf{m^3n^{11}}\end{aligned}$$

This shows multiplication of the same bases, so, all similar bases will be put together adding their indices.

Example 4 Simplify $(2a^4)(a^2b^3)$

Solution:

$$\begin{aligned}(2a^4)(a^2b^3) &= 2(a^{4+2})(b^3) \\ &= \mathbf{2a^6b^3}\end{aligned}$$

This is a multiplication of the same bases but only the indices of **a** is to be added.

Example 5 Simplify $12p^4q \div 6q$

Solution:

$$\begin{aligned}12p^4q \div 6q &= (12 \div 6)(p^4)(q^{1-1}) \\ &= 2p^4q^0 \\ &= \mathbf{2p^4}\end{aligned}$$

This is a combination of the division rule and zero index rules. All zero indices must be simplified.

Example 6

Simplify $\frac{10rs^5}{2r^3s^2}$

Solution:

$$\begin{aligned}\frac{10rs^5}{2r^3s^2} &= (10 \div 2)(r \div r^3)(s^5 \div s^2) \\ &= 5r^{1-3}s^{5-2} \\ &= 5r^{-2}s^3 \\ &= \frac{5s^3}{r^2}\end{aligned}$$

This is a combination of the division rule and the negative index rule. All negative indices must be simplified if possible to positive index.

$$\text{i.e. } r^{-2} = \frac{1}{r^2}$$

Example 7 Simplify $\frac{(3p^2q)^2}{p^2q^3}$

Solution:

$$\begin{aligned}\frac{(3p^2q)^2}{p^2q^3} &= \frac{3^2p^{2 \times 2}q^{1 \times 2}}{p^2q^3} \\ &= \frac{9p^4q^2}{p^2q^3} \\ &= 9p^{4-2}q^{2-3} \\ &= 9p^2q^{-1} \\ &= \frac{9p^2}{q}\end{aligned}$$

This is a combination of the division rule, power and negative index rules.

The Power rule has to be used first for the expression inside the parentheses, then division.

NOW DO PRACTICE EXERCISE 12

**Practice Exercise 12**

1. Evaluate:

(a) 273^1

(b) $(2^3)^4$

(c) 101^0

(d) $5^2 - 5^0$

(e) $(-2)^4$

(f) $3 + 3^2 + 3^3$

2. Simplify the following expressions with no zero and negative index.

(a) $3a^2b \times 2a^4b^3$

(b) $3(p^2)^4$

(c) $\frac{16xy^3}{2y}$

(d) $\frac{36r^3s^3}{12rs^2}$

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 2

TOPIC 2: SUMMARY



This summarizes some of the important ideas and concepts to remember.

Index Laws

The manipulation of powers, or indices or exponents is a very critical and underlying skill to have in algebra. In essence there are just 3 basic laws and from those we can derive other interesting and useful rules.

- The three basic rules are:

Multiplication Rule	$a^m \times a^n = a^{m+n}$
Division Rule	$a^m \div a^n = a^{m-n}$
Power Rule	$(a^m)^n = a^{m \times n}$

- Other rules that are derived are:

Zero Index Rule	$a^0 = 1$
Negative Index Rule	$a^{-m} = \frac{1}{a^m}$

Scientific Notation

- You can write very large or very small numbers more simply using scientific notation or standard notation.

Scientific form is always written as $a \times 10^b$ where a is a number between 1 and 10, but never equal to 10.

- To convert a number quickly to scientific notation form or standard form, move the decimal point either to the left (if whole number) or to the right (if decimal number) arriving at $a < 10$ and the number is then written as a product of a number between 1 and 10 and a power of 10. The Standard form can also be converted back to an ordinary number by reversing the procedure in converting a number to scientific notation form or standard form.

REVISE LESSONS 7- 12. THEN DO TOPIC TEST 1 IN ASSIGNMENT BOOK 2.

ANSWERS TO PRACTICE EXERCISES 7-12

Practice Exercise 7

1. (a) 289 (b) 225 (c) 0.49 (d) 0.1225 (e) $\frac{4}{9}$
 2. (a) ± 8 (b) ± 9 (c) ± 4 (d) ± 13
 3. (a) 85 (b) 392
 4. (a) 3 and 4 (b) 4 and 5 (c) 2 and 3
-

Practice Exercise 8

1. (a) m^5 (b) a^7
 2. (a) M^8 (b) t^{19} (c) v^{13}
 3. (a) 5^8 (b) 17^{18} (c) k^{16} (d) z^{60}
 4. (a) 6^9 (b) 5^{14} (c) a^8 (d) m (e) 4^2
-

Practice Exercise 9

1. (a) 7^5 (b) 3^1 (c) 8^3
 2. (a) h^5 (b) $4f^1$ (c) $3y^7$ (d) $A^0 = 1$
 3. (a) 12^4 (b) 4^2 (c) e^3 (d) h^8 (e) 15^1
-

Practice Exercise 10

1. (a) 1 (b) 1 (c) 3 (d) 2
 2. (a) $\frac{1}{t^{10}}$ (b) $\frac{3}{k^1}$ (c) $\frac{1}{(2y)^5}$ (d) $\frac{3}{t^4}$
 3. (a) $x^0 = 1$ (b) $2y^0 = 2$ (c) m^{-4} or $\frac{1}{m^4}$ (d) x^{-9} or $\frac{1}{x^9}$
-

Practice Exercise 11

1. (a) 1.29×10^5 (b) 2.54×10^{-9}
2. (a) 78 300 000 km (b) 0.000 000 002 5 hours
3. (a) 4.97×10^3 (b) 9.31×10^6

4. (a) 8×10^{-2} (b) 5.29×10^{-4} (c) 4.26×10^{-1}
5. (a) 87.4 (b) 1075
(c) 0.000 005 (d) 0.0063
6. 400 days
-

Practice Exercise 12

1. (a) 273 (b) 4096 (c) 1
(d) 24 (e) 16 (f) 39
2. (a) $6a^6b^4$ (b) $3p^8$ (c) $8xy^2$ (d) $3r^2s$
-

END OF TOPIC 2

TOPIC 3

ALGEBRAIC EXPRESSIONS

Lesson 13: Number Patterns

Lesson 14: Algebraic Expressions

Lesson 15: Simplifying Algebraic Expressions

Lesson 16: Evaluating Algebraic Expressions

Lesson 17: Factorisation

TOPIC 3: ALGEBRAIC EXPRESSIONS

Introduction



Welcome to Topic 3 of Unit 2. In this topic, we will do the introductory lessons in Algebra which are number patterns and algebraic expressions.

Mathematics is especially useful when it helps you to predict and work with relationships of different quantities.

Number patterns are all about predictions and working with them leads directly to the concept of **algebraic expressions** and functions in mathematics which is a formal description of the relationships among different quantities.

Algebra is a branch of mathematics which encompasses every part of our lives. It is an important language of mathematics which allows us to communicate concepts.

In this Topic you will learn:

- how to recognize number patterns, how the terms of a number pattern are related and how to obtain the next term of each number pattern. You will also learn to identify the rules or formula used to work out any term of a sequence without working out all the terms.
- translate mathematical words, phrases and sentences into algebraic words, phrases and expressions
- simplify algebraic expressions by removing grouping symbols and collecting like terms
- evaluate algebraic expressions
- factorize algebraic expressions

Lesson 13: Number Patterns



Welcome to Lesson 13. This is the first lesson of Topic 3.

In this lesson, you will:

- define number patterns
 - identify the formula for arithmetic sequence and arithmetic series
 - find the missing terms in a sequence
 - find the sum of the n th terms of an arithmetic series
-

Recognizing number patterns is an important problem-solving skill. If you see a pattern when you look systematically at specific examples, you can use that pattern to generalize what you see into a broader solution to a problem.

Consider this question.

What do snowflakes, beehives and strands of DNA have in common?

This may seem like a strange question, but each of these involves a pattern.

In Mathematics, patterns involve some type of regularity. You can see different types of regularity.

For example: regularity of shape, direction, size or number relationships.

Patterns are important in mathematics because they can be extended, repeated or built upon and this is a key part to solving many problems.

People often use patterns to understand things. Numbers sometimes occur in patterns.

What are Number patterns?



Number patterns are list of numbers that follow a certain sequence or pattern.

You have already seen the following number patterns.

2, 4, 6, 8 ... even numbers

1, 3, 5, 7 ... odd numbers

5, 10, 15, 20 ... multiples of 5

Remember:

- A pattern shows some kind of regularity in shape, size or direction.
- When numbers form pattern, you can predict the next number using a rule.
- A list of numbers which form a pattern is called a **sequence**.
- Each number of a pattern is called a **term**.

Example 1

Find a rule for predicting the next number in each of the following patterns and use the rule to find the missing numbers.

a) 4, 7, 10, 13, 16 ____, ____, ____

b) 3, 12, 48, 192, ____, ____, ____

Solution

- a) Compare each term with the term before and describe what you see.

Each term is 3 more than the term before it.

Write these as a rule.

Add 3 to any term in the pattern to find the next term.

Use the rule to find the missing numbers.

4, 7, 10, 13, 16, 19, 22, 25

- b) Compare each term with the term before and describe what you see.

Each term is 4 times the term before.

Write these as a rule.

Multiply any term in the pattern by 4 to find the next term.

Use the rule to find the missing numbers.

3, 12, 48, 192, 768, 3072, 12 288

Sometimes, the rules for patterns involve more than one operation.

Example 2

Consider some patterns defined by the following sets:

1. {1, 3, 5, 7, ...}
2. {1, 4, 9, 16, ...}
3. {2, 9, 10, 13, ...}

Note that if we take each set as consisting of so many elements, it will be difficult to name all the elements. Hence, instead of listing the elements, we can just look for patterns described by the elements. We can easily write a general expression for the elements.

1. $\{2(1) - 1, 2(2) - 1, 2(3) - 1, 2(4) - 1, \dots\}$
2. $\{1^2, 2^2, 3^2, 4^2, \dots\}$
3. $\{1^3 + 1, 2^3 + 1, 3^3 + 1, 4^3 + 1, \dots\}$

Looking for patterns in the preceding numerical expressions, we observe that if we take $n = 1, 2, 3, \dots$ then, the elements of each set can be generalized as follows:

1. $2n - 1 \longrightarrow$ powers of 2
2. $n^2 \longrightarrow$ squares of natural numbers
3. $n^2 + 1 \longrightarrow$ cubes of positive integers increased by 1

What values will you obtain if you replace n by 5, 10 and 100 in each of the above?

Notice that when different values are substituted for n in each case, we obtain different elements. Thus, we refer to n as a variable or a pronumeral.

A variable or a pronumeral is a symbol used to represent a definite but unspecified number or numbers. Here we shall use letters to represent variables.

Common Number Patterns

Numbers can have interesting patterns. Here we list the most common patterns and how they are made.

1. Arithmetic Sequences

An **arithmetic sequence** is a sequence wherein the difference between one term and the next is always the same. This difference is called the “**common difference**”. The common difference is added to each term to get the next term.

Example 3

$$\begin{array}{ccccccccccc}
 1, & 4, & 7, & 10, & 13, & 16, & 19, & 22, & 25, & \dots \\
 \swarrow & \swarrow & \swarrow & \swarrow & \swarrow & \swarrow & \swarrow & \swarrow & \swarrow & \\
 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 &
 \end{array}$$

This sequence has a common difference of 3 between one term and the next.

The pattern is continued by **adding 3** to the last number each time.

So 3 must be added to obtain the next term in the sequence. The next two terms are

$$25 + 3 = 28$$

And $28 + 3 = 31$

Giving 1, 4, 7, 10, 13, 16, 19, 22, 25, 28, 31,

Example 4

3, 8, 13, 18, 23, 28, 33, 38, ...

This sequence has a common difference of 5 between one term and the next.

The pattern is continued by **adding 5** to the last number each time.

What is the common difference in this example?

19, 27, 35, 43, ...

Answer: The common difference is **8**

The common difference could also be negative, like this:

25, 23, 21, 19, 17, 15, ...

This common difference is **-2**

The pattern is continued by **subtracting 2** each time.

2. Geometric Sequences

A **geometric sequence** is a sequence wherein the ratio of one term to the next is always the same. This ratio is called the "**common ratio**". The common ratio is multiplied by each term to get the next term.

Example 5

2, 4, 8, 16, 32, 64, 128, 256, ...

This sequence has a common ratio of 2 between each term.

The pattern is continued by **multiplying by 2** each time.

Example 6

3, 9, 27, 81, 243, 729, 2187, ...

This sequence has common ratio of 3 between each term.

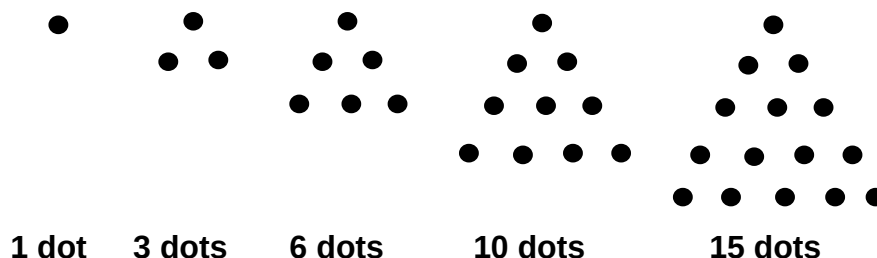
The pattern is continued by **multiplying by 3** each time.

3. Special Sequences**(a) Triangular Numbers**

1, 3, 6, 10, 15, 21, 28, 36, 45, ...

This triangular number sequence is generated from a pattern of dots which form a triangle.

By adding another row of dots and counting all the dots we can find the next number of the sequence:



(b) Square Numbers

1, 4, 9, 16, 25, 36, 49, 64, 81, ...

The next number is made by squaring where it is in the pattern.

The second number is 2 squared (2^2 or 2×2).

The seventh number is 7 squared (7^2 or 7×7) etc.

(c) Cube Numbers

1, 8, 27, 64, 125, 216, 343, 512, 729, ...

The next number is made by cubing where it is in the pattern.

The second number is 2 cubed (2^3 or $2 \times 2 \times 2$).

The seventh number is 7 cubed (7^3 or $7 \times 7 \times 7$) etc.

(d) Fibonacci Numbers

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, ...

The Fibonacci Sequence is found by adding the two numbers before it together.

The 2 is found by adding the two numbers before it ($1+1$).

The 21 is found by adding the two numbers before it ($8+13$).

The next number in the sequence above would be 55 ($21+34$).

Can you figure out the **next** few numbers?

NOW DO PRACTICE EXERCISE 13

**Practice Exercise 13**

Refer to the number pattern below to answer Questions 1 to 3.

6, 7, 8, 9,

1. The 5th term of the pattern is
 - (a) 5
 - (b) 8
 - (c) 10
 - (d) 30
 2. A rule that could be used to find each term given its position number is
 - (a) Position number + 5
 - (b) Position number + 6
 - (c) Position number $\times$ 6
 - (d) Position number $\times$ 6 + 1
 3. The 100th term of the pattern is:
 - (a) 105
 - (b) 106
 - (c) 600
 - (d) 109
-

Refer to the number pattern below to answer Questions 4 to 6.

5, 8, 11, 14, ...

4. The 5th term in the sequence is
 - (a) 15
 - (b) 16
 - (c) 17
 - (d) 18
 5. Which of the following rules could be used to find the numbers in this pattern?
 - (a) $3 \times \text{position number} + 2$
 - (b) $3 \times (\text{position number} + 1) - 1$
 - (c) Position number + 4
 - (d) $3 \times (\text{position number} - 1) + 5$
 6. The 100th term in the pattern is:
 - (a) 100
 - (b) 104
 - (c) 300
 - (d) 302
-
7. Which of the following number sentences is **not** true?
 - (a) $10 + 8 = 5 \times 3$
 - (b) $12 - 4 = 5 + 3$
 - (c) $16 \div 4 = 10 - 6$
 - (d) $4 \times 3 = 15 - 3$

8. Given $\Delta = 40$, which of the following number sentences is true?

- (a) $\Delta - 20 = 6 \times 4$
 - (b) $\Delta \times 2 = 5 \times 8$
 - (c) $6 \times 8 = \Delta + 8$
 - (d) $5 \times 9 = \Delta + 7$
-

9. How do you get back to the number you started with if you subtract 10 from it?

- | | |
|--------------------|------------------|
| (a) Add 10 | (b) Subtract 10 |
| (c) Multiply by 10 | (d) Divide by 10 |
-

10. When I divide a number by 3 the answer is 6. What is the number?

- | | |
|--------|-------------------|
| (a) 2 | (b) $\frac{1}{2}$ |
| (c) 18 | (d) 9 |
-

11. When I add 76 to a number the answer is 99. A number sentence that matches this statement is

- a. $\Delta = 99 + 76$
 - b. $\Delta - 76 = 99$
 - c. $\Delta + 76 = 99$
 - d. $\Delta + 99 = 76$
-

12. When I double a number then subtract 5, the answer is 11. The number I am thinking of is:

- | | |
|--------|-------|
| a. 32 | (b) 8 |
| (c) 12 | (d) 3 |
-

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 3.
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Lesson 14: Algebraic Expressions



You learnt what number patterns are in the last lesson. You also learnt the difference between arithmetic and geometric sequences



In this lesson, you will:

- translate math words, phrases and sentences to algebraic expressions
- write sentences into mathematical equations

Word sentences need to be turned into algebraic expressions so that we can do mathematical computations on them, and work out their numerical answers. Translating word problems into Algebra requires us to know how various words translate into mathematical symbols such as $+$, $-$, $\times$ and $\div$.

If the word problems only have numbers, then it is easy to translate them, once we have practiced doing a few examples.

Algebraic Expressions

An algebraic expression is made up of numbers and letters (or “pronumerals”), as well as $+$, $-$, $\times$ and $\div$ operation symbols.

Examples of algebraic expressions are:

$$y + 7$$

$$2m$$

$$3g - 4k$$

$$5 - 2L$$

$$3k / 2r$$

Mathematicians rewrite word situations into algebraic expressions to make them simpler and more direct to work with.

Algebraic expressions are used to make descriptions of mathematical situations a lot shorter. For example, it is easier to write “mv” than “mass x velocity”.

Think of it as like this:

On a calculator it would be impossible to type in words:

“Six times twelve divided by three” and it is much easier to type in $6 \times 12 \div 3$

Translation Tips for Word Problems

The following translation tables help convert word problems into algebraic expressions. Look for the following key words and phrases when reading through word problems.

Whenever you find one of these words, translate the word or phrase into $+$, $-$, $\times$, $/$, or $=$.

This will help form the Algebra equation for the word problem.

- The following words in a sentence indicate that Addition and Subtraction are taking place.

Words for Addition

plus
sum
and
total
combined
still
perimeter
added to
more than
increased by
next year
longer than
gain
together
the sum of
combined together
heavier than
the total of
older than
bigger than

Words for Subtraction

less
difference
removed
separated
fewer
less
subtracted
less than
decreased by
last year
shorter than
smaller than
differ by
subtracted from
taken away
lighter than
the difference of
younger than
how many more

We need to be careful when translating subtraction words!!!

“Five dollars less than twenty dollars is fifteen dollars”.

Five dollars less than twenty does **NOT** translate to $5 - 20$.

Correct translation:

“less than” **swaps** the order of numbers

$$20 - 5 = 15$$

- The following words in a sentence indicate that **division** and **multiplication** are taking place.

Words for Division

per
out of
over
split
quotient
ratio
halved
half
split so many ways
divided by
fraction into
equal parts

Words for Multiplication

times
of
product
area
multiplied by
doubled
tripled
twice as
three times as
quadrupled
volume

The following words in a sentence need to be translated into an **equal** sign.

Words for Equals		
equals	results in	to obtain
gives	will be	has a value of
was	makes	is the same as
is	becomes	is equivalent to

Whenever we get a word problem to convert into Algebra, we can use these translation tables to help us work out what mathematical symbols we need to use to replace the words.

Here are some simple examples.

Mathematical Statement	Translation
a) the sum of ten and y	$\longrightarrow 10 + y$
b) four less than x	$\longrightarrow x - 4$
c) x multiplied by 22	$\longrightarrow 22x$
d) the quotient of n and 3	$\longrightarrow n/3$
e) two less than the total of a number and five	$\longrightarrow n + 5 - 2$

Example 1

Alex has C chocolates, and D drinks. Miley has four fewer chocolates, but twice as many drinks. Write an expression for Miley's snacks.

Solution:

Replace the words with numbers and Maths symbols:

Four $\longrightarrow 4$
 Fewer chocolates $\longrightarrow C - 4$ (Remember minus REVERSES)
 Twice as many drinks $\longrightarrow 2D$
 Answer : $C - 4 + 2D \longrightarrow$ **Final answer: $C - 4 + 2D$**

Example 2

In a fruit bowl there are "a" apples and "b" bananas. In a paper bag there are 6 apples and 8 bananas. What is the total number of pieces of fruit?

Solution:

Replace the words with numbers and maths symbols:

"a" apples and "b" bananas $\longrightarrow a + b$
 6 apples and 8 bananas $\longrightarrow 6 + 8$ (not $6a + 8b$)

Combining our two expressions we have:

$a + b + 6 + 8 \longrightarrow$ **Final Answer : $a + b + 14$**

Example 3:

Brad takes “h” hours and “m” minutes to complete a mini triathlon. His friend Leonardo takes twice as long to finish the race. Write an algebraic expression for Leo’s race time.

Solution:

Replace the words with numbers and maths symbols:

“h” hours and “m” minutes $\longrightarrow$ $h + m$

Twice “h” hours and “m” minutes $\longrightarrow$ $2(h + m)$

$2(h + m) \longrightarrow 2(h) + 2(m) \longrightarrow$ **Final Answer: $2h + 2m$ (not $2h + m$)**

NOW DO PRACTICE EXERCISE 14

**Practice Exercise 14**

1. Write an equation for each of the following. Let x be the unknown number.
 - a) Five is added to a number to give a result of twelve
 - b) Ten is subtracted from a number and the result is four.
 - c) A number multiplied by four gives eight.
 - d) A number divided by nine is equal to six.

2. Choose the correct equation that describes each of the following sentences:
 - a) Three is added to a number (x) to give a result of seven.
 - A. $3 + 7 = x$
 - B. $x + 3 = 5$
 - C. $x + 3 = 7$
 - D. $x + 7 = 3$
 - b) A number (a) is subtracted from five to give a result of two.
 - A. $a - 5 = 2$
 - B. $a = 2 - 5$
 - C. $5 - a = 2$
 - D. $a - 2 = 5$
 - c) Twice a number (x) more than four is equal to eighteen.
 - A. $2x = 18$
 - B. $4 + 2x = 18$
 - C. $4x + 2 = 18$
 - D. $2(x+4) = 18$

3. Write an equation for each of these sentences. Let k be the unknown number.
 - a) A number is multiplied by three, and then two is added to give a result of eleven.

- b) A number is divided by four, then one is subtracted to give a result of three
- c) The sum of a number and two divided by six is seven.
- d) One subtracted from a number multiplied by eight gives sixteen.
- e) A number has been tripled and seven is added to give a result of nineteen.

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 3

Lesson 15: Simplifying Algebraic Expressions



You learnt about the different types of number patterns in the last lesson.



In this lesson, you will:

- define simplifying algebraic expressions
- simplify expressions by collecting like terms
- simplify expressions by removing grouping symbols

In algebra, we often get very long expressions that we need to make simpler. Simpler expressions are easier to solve.

If you have studied Grade 8 Mathematics in FODE, you would have learned the meaning of **simplifying algebraic expressions**. In case you have forgotten, below is the meaning again.

Simplifying algebraic expression means to make an expression simple in the sense that the expression has fewer operation signs, parentheses or other grouping symbols.

In this lesson you will extend further your knowledge and skills in simplifying algebraic expressions by collecting like terms. This skill is very important when solving equations. We begin by defining like terms.

- Terms are said to be **like or similar terms**, if they have the variables (letters or pronumerals) raised to the same exponent.
- Terms with different or the same variables but raised to different exponents are said to be **unlike or dissimilar terms**.

Example 1

- (a) $5x^2$, $-2x^2$ and x^2 are like terms
- (b) $3ny$, $-2n$, $4y$ are unlike terms because their literal parts are different
- (c) $\frac{1}{5}yx$, $\frac{1}{2}xy$, $2xy$ are like terms
- b. $6x^4$, $7y^4$, $5z^2$ are unlike terms
- c. $6ab$, $3ac$, $4bc$ are unlike terms
- d. $7mn^4$, $10mn^4$, $20mn^4$ are like terms

Like terms can be added .

For example:

Consider $5x + 2x$

$5x$ means $x + x + x + x + x$ and similarly $2x$ means $x + x$

Thus, $5x + 2x = x + x + x + x + x + x + x = 7x$

Note that 7 is the sum of the numerical coefficients.

Now consider $5x - 2x$. In this case $5 - (+2) = 3$

Thus , $5x - 2x = 3x$

The above 2 examples suggest the following steps or rules for adding like terms.

Step 1 Identify the terms with the same literal parts. These are the like terms.

Step 2 Add the numerical coefficients of like terms.

Step 3 Copy the same literal part and attach to the sum of the numerical coefficient

This is sometimes called **collecting** or **combining** like terms .

Simplifying algebraic expressions means collecting or combining like terms to make a single expression. Collecting or combining like terms can be done by adding or subtracting them.

Collecting like terms

To simplify an expression, we collect **like terms**.

Example 1

Rewrite $2a + 4a$ as a single expression

Solution:

- 1.The terms $2a$ and $4a$ are like terms . The common pronumeral is a
2. The coefficients of the like terms are 2 and 4. Hence, $2 + 4 = 6$
3. Attach the common pronumeral to the sum of the numerical coefficients.

$$2a + 4a = 6a$$

Example 2

Simplify $4y^2 + 2y^2 + 3x - 2x$

Solution:

Collect like terms. $4y^2 + 2y^2$ and $3x - 2x$ are like terms.

The coefficients of the like terms are $4 + 2 = 6$ and $3 - 2 = 1$

Attach the common pronumeral to each of the sum of the numerical coefficients .

$$\begin{aligned} 4y^2 + 2y^2 + 3x - 2x &= 6y^2 + 1x \\ &= 6y^2 + x \end{aligned}$$

Example 3

Now look at the expression: **$4x + 5x - 2 - 2x + 7$**

To simplify: The x terms can be collected together to give 7x.

The numbers can be collected together to give 5.

So $4x + 5x - 2 - 2x + 7$ simplified is $7x + 5$.

Remember that terms are separated by + and - signs, and these are always written before a term.

Here is another example.

Example 4

Simplify this expression: **$x + 5 + 3x - 7 + 9x + 3 - 4x$**

Solution:

Write down the expression $x + 5 + 3x - 7 + 9x + 3 - 4x$

Collect all the terms together which are alike. Remember that each term comes with an operation (+, -) which goes before it.

$$x + 3x + 9x - 4x + 5 - 7 + 3$$

Simplify the x terms. $x + 3x + 9x - 4x = 9x$

Simplify the numbers separately. $5 - 7 + 3 = 1$

Answer: $x + 5 + 3x - 7 + 9x + 3 - 4x$ can be simplified to **$9x + 1$**

Different terms

There are times when you will have to simplify an expression that has many different terms or letters.

Have a look at this typical example. You will notice that there are different letters in this expression: x , y and z .

Example 5

Simplify the expression $5x + 3y - 6x + 4y + 3z$

Solution:

Write down the expression	$5x + 3y - 6x + 4y + 3z$
Collect the like terms together	$5x - 6x + 3y + 4y + 3z$
Simplify your expression, x first	$5x - 6x = -x$ (ie, $-1x$)
Then simplify y	$3y + 4y = 7y$
Then simplify z	$3z$
The answer is:	$-x + 7y + 3z$

Simplifying Expressions by Removing Grouping Symbols

Earlier, we simplified expressions either by adding or subtracting like terms. Another way of simplifying expressions is by removing grouping symbols. The rule for adding like terms is based on an important property called the distributive property of multiplication over addition.

$a(b + c) = ab + ac$ $(b + c)a = ba + ca$

Example 6

Use the distributive property to remove the grouping symbol.

$$2(3x^2 + 1) = 2(3x^2) + 2(1)$$

Distribute 2 by multiplying each of the term inside the parenthesis.

$$= 6x^2 + 2$$

Example 7

$$-4(2x - 3) = -4(2x) - (-4)(3)$$

Distribute (-4) by multiplying to each of the term inside the parentheses.

$$= -8x - (-12)$$

Rules on multiplication of directed numbers apply.

$$= -8x + 12$$

If you see an addition or subtraction problem inside a set of parenthesis, you must use the distributive property BEFORE simplifying the expression.

As you look through the next 2 examples, notice how the distributive property was used first, then the algebraic expression was simplified.

Example 8

$$2(x + 4) + 3(x - 5) - 2y$$

Solution:

$$2x + 8 + 3x - 15 - 2y$$

Use the distributive property to get rid of the parenthesis

$$2x + 3x - 2y + 8 - 15$$

Rewrite with like terms together. Don't forget to take the sign before the term.

$$5x - 2y - 7$$

Simplify

Example 9

$$3(ab - a) - 2(b - a) - 4ab$$

Solution:

$$3ab - 3a - 2b + 2a - 4ab$$

Use the distributive property to get rid of the parenthesis. Don't forget to distribute the 2 as -2 since the subtraction sign is in front of it!

$$3ab - 4ab - 3a + 2a - 2b$$

Rewrite with like terms together. Don't forget to take the sign before the term.

$$-1ab - 1a - 2b$$

Simplify

$$-ab - a - 2b$$

No need to write 1 as a coefficient.

A term made up of variables or letters only is understood to be having a numeral coefficient of 1.

Understanding the examples given so far, you should be ready to try some on your own.

NOW DO PRACTICE EXERCISE 15

**Practice Exercise 15**

1. Simplify, where possible, by collecting like terms.

a) $4a + 5a$

b) $2x + x$

c) $17y - y$

d) $3b^2 - b^2$

e) $yx + 2xy$

f) $6x - 3x + 7x$

g) $7ab - 2ab + 11ab$

h) $9a^2 + 13a^2 - 17a^2$

2. Expand the following expressions:

a) $5(d + 4)$

b) $6(d - 3)$

c) $-3(a + 2)$

d) $-x(x - 4)$

e) $-p(p + q)$

3. Expand and simplify by collecting like terms.

a) $4(x + 3) - 2(x - 1)$

b) $3a - 7(a - 6) + 20$

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 3

Lesson 16: Evaluating Algebraic Expressions



In the previous lesson, you learnt to simplify algebraic expressions by collecting like terms and removing grouping symbols.



In this lesson, you will:

- define evaluating algebraic expressions
 - enumerate the steps in evaluating expressions
 - evaluate algebraic expressions
-

Now you learnt how to simplify algebraic expressions, in this lesson you will learn how to evaluate algebraic expressions.

You have learned that in an algebraic expression, letters can stand for numbers.

When substituting or replacing a specific value for each **variable** in an expression, and then perform the operations, it is called **evaluating** the expression.

Hence,

Evaluating algebraic expressions is the process of substituting or replacing a specific value for each variable or letter in an expression and then simplifying by performing the operations.

In order to evaluate an algebraic expression, you must know the exact values for each variable. Then you will simply substitute and evaluate using the **order of operations**.

Take a look at the following examples:

Example 1 Evaluate the expression : $3s^2 + 1$, when $s = 2$.

Solution:

$$3s^2 + 1 = 3(2)^2 + 1$$

Substitute 2 for s in the expression

$$= 3(4) + 1$$

Using the order of operations, we must first evaluate the exponents

$$= 12 + 1$$

perform the multiplication

$$= 13$$

perform the addition

Therefore, **13 is the Final Answer**

Now, let's evaluate algebraic expressions with more than one variable. Don't forget to always use the **order of operations** when evaluating the expression after substituting.

Example 2 Evaluate the expression $\frac{2x^2y}{x-y}$ when $x = 5$ and $y = 1$.

Solution:

$$\begin{aligned}\frac{2x^2y}{x-y} &= \frac{2(5)^2(1)}{5-1} && \text{Substitute the given values for each variable.} \\ &= \frac{2(25)(1)}{5-1} && \text{Evaluate the numerator first starting with the exponent} \\ &= \frac{50}{4} && \text{Perform the operations in the numerator and the denominator} \\ &= 12\frac{2}{4} && \text{Perform the division} \\ &= 12\frac{1}{2} \text{ or } 12.5 && \text{Simplify the fraction.}\end{aligned}$$

Therefore, $12\frac{1}{2}$ or **12.5 is the final answer.**

In the next example we will look at the fraction bar as a grouping symbol and evaluating the expression when you have more than one variable.

Example 3 Evaluate the expression $a + \frac{(3+b^3)}{2} - c$, when $a = 4$, $b = 3$, $c = 8$.

Solution:

$$\begin{aligned}a + \frac{(3+b^3)}{2} - c &= 4 + \frac{(3+3^3)}{2} - 8 && \text{Substitute the given values for each variable} \\ &= 4 + \frac{(3+27)}{2} - 8 && \text{Start inside the parenthesis and evaluate the power (exponent)} \\ &= 4 + \frac{30}{2} - 8 && \text{Add 3 and 27 to get 30.} \\ &= 4 + 15 - 8 && \text{Divide 30 by 2 to get 15.} \\ &= 19 - 8 && \text{Addition and subtraction are left, but addition comes first.} \\ &= 11 && \text{subtract 8 from 19 to get 11.}\end{aligned}$$

Therefore, **11 is the final answer.**

Example 4 Find the value of the expression $\frac{3xy}{2x - 4y}$ when $x = 3$ and $y = -4$.

Solution: In this expression, we will compute the numerator and denominator separately, then simplify the resulting fraction.

$$\begin{aligned}
 \frac{3xy}{2x - 4y} &= \frac{3(3)(-4)}{2(3) - 4(-4)} && \text{substituting the given values for variables} \\
 &= \frac{-36}{6 - (-16)} && \text{computing the multiplication} \\
 &= \frac{-36}{6 + 16} && \text{re-writing subtraction as addition} \\
 &= \frac{-36}{22} && \text{adding} \\
 &= \frac{-36 \div 2}{22 \div 2} && \text{factoring the 36 and 22 by their GCF} \\
 &= -\frac{18}{11} && \text{simplifying fractions}
 \end{aligned}$$

Therefore, the value of $\frac{3xy}{2x - 4y}$ when $x = 3$ and $y = -4$ is $-\frac{18}{11}$.

Example 5 Find the value of $\frac{6xy}{x^2 - y^2}$ when $x = -4$ and $y = 2$.

Solution:

$$\begin{aligned}
 \frac{6xy}{x^2 - y^2} &= \frac{6(-4)(2)}{(-4)^2 - (2)^2} && \text{substituting the for variables} \\
 &= \frac{6(-4)(2)}{16 - 4} && \text{computing the exponents} \\
 &= \frac{-48}{16 - 4} && \text{multiplying} \\
 &= \frac{-48}{12} && \text{subtracting} \\
 &= -4 && \text{simplifying}
 \end{aligned}$$

Therefore, the value of $\frac{6xy}{x^2 - y^2}$ when $x = -4$ and $y = 3$ is -4 .

If you are familiar with the **order of operations**, then evaluating algebraic expression is quite easy. Just remember to substitute the given values for each variable and simplify the resulting numerical expression.

NOW DO PRACTICE EXERCISE 16

**Practice Exercise 16**

1. Evaluate the expression $v = u + at$ when:

(a) $u = 4$, $a = 3$ and $t = 5$

13. $u = 8$, $a = 2$ and $t = 10$

2. The area of a triangle is given by $A = \frac{1}{2}bh$, where A is the area, b is the base of the triangle and h is the perpendicular height.

(a) Find the area of a triangle, A , when $b = 8$ m and $h = 7$ m.

(b) Find the height when $A = 30\text{cm}^2$ and $b = 5\text{cm}$.

3. The time T minutes to cook a joint of meat weighing w kg is $T = 45w + 20$.

(a) Find the time to cook a joint weighing 3 kg.

(b) Find the weight of the joint of meat, w , that took 110 minutes to cook.

4. The perimeter of a rectangle is given by $P = 2l + 2w$ where l is the length of the rectangle and w is the width. Find the perimeter of a rectangle if:

(a) $l = 5$ and $w = 4$

(b) $l = 8$ and $w = 10$

(c) $l = 7$ and $w = 12$

-
5. Evaluate the following when $a = 3$, $c = 2$ and $e = 5$

(a) $3a - 2$

(b) $4c + e$

(c) $2c + 3a$

(d) $5e - a$

(e) $e - 2c$

-
6. Evaluate the following when $h = 3$, $m = -2$ and $t = -3$

(a) $2m - 3$

(b) $4t - 10$

(c) $3h - 12$

(d) $6m + 4$

(e) $9t - 3$

-
7. When $a = -4$ and $b = 2$, find the value of:

(a) $4a^3 - 3b^2$

(b) $\frac{7ab}{2a + 2b}$

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 3

Lesson 17: Factorization



You have learned to evaluate algebraic expressions by collecting terms and by removing grouping symbols in Lesson 16.



In this lesson, you will:

- define factorization
- identify common monomial and polynomial factors
- factorise expressions to its simplest form

In your early study of mathematics you learnt about the meaning of factors and factorization.

Numbers have factors.

For example: The factors of 12 are 1, 2, 3, 4, 6 and 12.

We say, $12 = 1 \times 12$,
 $12 = 2 \times 6$ and
 $12 = 3 \times 4$,

We can also say, $12 \div 12 = 1$
 $12 \div 3 = 4$ and
 $12 \div 6 = 2$

In algebra, expressions also have factors.

For example: The factors of the expression $2x + 2y$ are 2 and $x + y$.

We say, $2x + 2y = 2(x + y)$

Also we say $(2x + 2y) \div 2 = x + y$
 $(2x + 2y) \div (x + y) = 2$

In this lesson, you will again look through the meaning of factorization.

- **Factorization** (also **factorisation** in British English) or **factoring** is the process of finding the factors which when multiplied together gives the original number. It is a way of expressing a number or expression as product of certain factors.
- A factor is a number or term that divides exactly into another number or term.

Factorization is like splitting a number or an expression into a product of simpler expressions.

For example

Factor $4x + 20$

Both $4x$ and 20 have a common factor of 4

$4x$ is $(4)(x)$

20 is $(4)(5)$

So you can factor the whole expression into:

$$4x + 20 = 4(x+5)$$

So, $4x+20$ has been "factored into" 4 and $x+5$.

Using the Distributive property let us change the expression from a product to a sum by finding the GCF and Polynomial Factor.

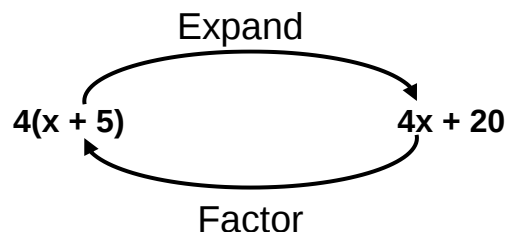
For example,

An expression such as $4m(x - b)$ tells you to multiply $4m$ by $x - b$. when you do that, you get the sum $4mx - 4mb$. When you do that in reverse, by writing $4mx - 4mb$ as the product of the two factors $4m$ by $x - b$, you are **factoring**.

Factoring is simply the process of finding the factors of a given product.

To factorize an algebraic expression means to change a sum into a product. It is the opposite of expansion.

For example



You saw in the previous example that both $4x$ and 20 have a common factor of 4 .

But to do the job properly, make sure you have the **Highest Common Factor (HCF)**, including any variables.

Basically, when factoring algebraic expressions, you will first look for the HCF and use your HCF to make your expression looks like a multiplication problem:

Example

Factor $3y^2 + 15y$

Firstly, 3 and 15 have a common factor of 3 .

So you could have:

$$3y^2 + 15y = 3(y^2 + 5y)$$

But we can do better!

$3y^2$ and $15y$ also share the variable y . Together that makes $3y$ which is the **Highest Common Factor** $3y^2$ and $15y$.

$$3y^2 \text{ is } 3y(y)$$

$$15y \text{ is } 3y(5)$$

So you can factor the whole expression into: $3y^2+12y = 3y(y+5)$

Check: $3y(y+5) = 3y(y) + 3y(5) = 3y^2+15y$

We call $3y$ as the **common monomial factor** and $y + 5$ the **common polynomial factor**.

- **A Common Factor is a term that occurs as a factor in each of the expressions being added or subtracted.**
- **A common monomial factor is the product of the literal and numerical factor common to all terms of the polynomial.**
- **The Highest Common Factor is the highest among the common factors in each expressions being added or subtracted.**

Now study the following examples of factorisation of algebraic expression.

Example 1

Factorize $8x^2 + 28x$

Solution: Both 8 and 28 have a common factor of 4 and they also share a variable x .

Together that makes $4x$ which is the **HCF** of $8x^2$ and $28x$.

So you can factor $8x^2 + 28x$ by dividing by $4x$

$$\text{Therefore, } 8x^2 + 28x = 4x(2x + 7)$$

Example 2

Factorize $2x^2 - 6x^3 + 8x^4$

Solution: $2x^2 - 6x^3 + 8x^4$ has $2x^2$ as a common factor,

$$\text{So, } 2x^2 - 6x^3 + 8x^4 = 2x^2(1 - 3x + 4x^2) \quad \text{divide each term by HCF}$$

Therefore, the factors of $2x^2 - 6x^3 + 8x^4$ are $2x^2$ and $(1 - 3x + 4x^2)$.

Check using the distributive law of multiplication.

$$\begin{aligned} 2x^2(1 - 3x + 4x^2) &= 2x^2(1) - 2x^2(3x) + 2x^2(4x^2) \\ &= 2x^2 - 6x^3 + 8x^4 \end{aligned}$$

Example 3

Factorize the expression: $2x^2y + xy^2 - x^2y^2$

Solution: $2x^2y$, xy^2 and x^2y^2 have both xy as Highest common factor (HCF)

divide each term by the HCF xy .

Therefore, the factors of $2x^2y + xy^2 - x^2y^2$ are xy and $(2x + y - xy)$.

Example 4

Factorize the expression $2x^3 - x^2 + x$

Solution:

The expression $2x^3 - x^2 + x$ has x as highest common factor (HCF)

Divide each term by the HCF x .

So, $2x^3 - x^2 + x = x(2x^2 - x + 1)$.

All the examples so far suggest the following steps that you have to follow when factoring algebraic expressions.

STEPS

1. Find the HCF of all the terms of the expression
2. Divide each term of the expression by the HCF.
3. The quotient and the HCF are the factors of the given expression.

NOW DO PRACTICE EXERCISE 17

**Practice Exercise 17**

1. List the factors of these numbers, then calculate the HCF.

a. 12 and 15

(b) 10 and 25

14. 20 and 16

15. 18 and 27

16. 14 and 28

17. 36 and 42

2. Complete by finding the HCF.

(a) $aq + ar = \underline{\hspace{2cm}} (q + r)$

(b) $kp - dp = \underline{\hspace{2cm}} (k - d)$

(c) $2a - 6 = \underline{\hspace{2cm}} (a - 3)$

(d) $8d + 16 = \underline{\hspace{2cm}} (d + 2)$

(e) $10k + 15 = \underline{\hspace{2cm}} (2k + 3)$

3. Complete by writing the correct terms in the brackets

(a) $wx - xk = x(\underline{\hspace{2cm}})$

(b) $pd - 5p = p(\underline{\hspace{2cm}})$

(c) $fw - wg = w(\underline{\hspace{2cm}})$

(d) $10 - 5b = 5(\underline{\hspace{2cm}})$

(e) $6p - 12q = 6(\underline{\hspace{2cm}})$

4. Factorise:

(a) $y^2 + y =$

(d) $14w + 21 =$

(b) $4n^2 - 2n =$

(e) $4d + dc =$

(c) $9h - 9 =$

(f) $5hg + g^2 =$

5. Factorise the following by taking out the negative common factor.

(a) $-10x + 5$

(b) $-3y - 9$

(c) $-20c + 15$

(d) $-xy + 4y$

(e) $-7q - q^2$

6. Factorise:

(a) $4x^3 + 2x^2 + 2x$

(b) $x^3 + x^2 + x$

(c) $15a + 12b + 6c$

(d) $9x^2 - 3x + 15x^3$

(e) $2ab^2 + ab^2c - 3ab$

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 3

TOPIC 3: SUMMARY



This summarizes the important ideas and concepts to remember.

- **Number patterns** are a list of numbers that follow a certain sequence or pattern.
A list of numbers which form a pattern is called a **sequence**.
- **A variable or a pronumeral** is a symbol or letter used to represent a definite but unspecified number or numbers
- **An Arithmetic sequence** is a sequence made by adding some constant value each time.
- The value added each time in a arithmetic sequence is called the **common difference**.
- **A Geometric sequence** is a sequence made by multiplying a term by some constant value each time to get the next.
- The value multiplied each time in a geometric sequence is called the **common ratio**.

There are special sequences we come across in mathematics. These are the:

- **Triangular numbers** which is a pattern generated from a pattern of dots which form a triangle.
- **Square Numbers** which is a pattern formed by squares of numbers.
- **Cube numbers** which is a pattern formed by cubes of numbers.
- **Fibonacci sequences which is** a pattern formed by adding the two previous terms to get the next term. [i.e. 1,1, $(1 + 1)$, $(2 + 1) + \dots$].
- An **algebraic expression** is made up of numbers, variables or combination of a number and variables connected or joined by signs of operations and grouping symbols.

REVISE LESSON 13-17 THEN DO TOPIC TEST 3 IN ASSIGNMENT BOOK 2.

ANSWERS TO PRACTICE EXERCISE 13 - 17

Practice Exercise 13

- | | |
|------|-------|
| 1. c | 7. a |
| 2. a | 8. c |
| 3. a | 9. a |
| 4. c | 10. c |
| 5. a | 11. c |
| 6. d | 12. b |
-

Practice Exercise 14

1. (a) $5 + x = 12$
(b) $x - 10 = 4$
(c) $4x = 8$
(d) $x \div 9 = 6$
2. (a) C (b) C (c) B
3. (a) $3k + 2 = 11$
(b) $k \div 4 - 1 = 3$ or $\frac{k}{4} - 1 = 3$
(c) $(2 + k) \div 6 = 7$ or $\frac{2 + k}{6} = 7$
(d) $8(k - 1) = 16$
(e) $3k + 7 = 19$
-

Practice Exercise 15

1. (a) $9a$ (b) $3x$ (c) $16y$ (d) $2b^2$
(e) $3xy$ (f) $10x$ (g) $16ab$ (h) $5a^2$
2. (a) $5d + 20$ (b) $6d - 18$ (c) $-3a - 6$
(d) $-x^2 + 4x$ (e) $-p^2 - pq$
3. (a) $4x + 12 - 2x + 2 = 2x + 14$
(b) $3a - 7a + 42 + 20 = -4a + 62$
-

Practice Exercise 16

1. (a) $v = u + at = 4 + 3(5) = 19$
(b) $v = u + at = 8 + 2(10) = 28$

2. (a) $A = \frac{1}{2} (b)(h) = \frac{1}{2} (8)(7) = 28\text{cm}^2$
 (b) $h = 12 \text{ cm}$
3. (a) $T = 45w + 20 = 45(3) + 20 = 155 \text{ minutes}$
 (b) $w = 2 \text{ kg}$
4. (a) $P = 18\text{cm}$
 (b) $P = 36 \text{ cm}$
 (c) $P = 38 \text{ cm}$
5. (a) 7 (b) 13 (c) 13
 (d) 22 (e) 1
6. (a) -7 (b) -22 (c) -3 (d) -8
 (e) -30
7. (a) -268 (b) 14
-

Practice Exercise 17

1. (a) 3 (b) 5 (c) 4
 (d) 9 (e) 14 (f) 6
2. (a) $a(q + r)$ (b) $p(k - d)$ (c) $2(a - 3)$
 (d) $8(d + 2)$ (e) $5(2k + 3)$
3. (a) $x(w - k)$ (b) $p(d - 5)$ (c) $w(f - g)$
 (d) $5(2 - b)$ (e) $6(p - 2q)$
4. (a) $y(y + 1)$ (b) $2n(2n - 1)$ (c) $9(h - 1)$
 (d) $7(2w + 3)$ (e) $d(4 + c)$ (f) $g(5h + g)$
5. (a) $-5(2x - 1)$ (b) $-3(y + 3)$ (c) $-5(4c - 3)$
 (d) $-y(x - 4)$ (e) $-q(7 + q)$
6. (a) $2x(2x^2 + x + 1)$
 (b) $x(x^2 + x + 1)$
 (c) $3(5a + 4b + 2c)$
 (d) $3x(3x - 1 + 5x^2)$
 (e) $ab(2b + bc - 3)$
-

TOPIC 4

EQUATIONS

- | | |
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| Lesson 18: | Simple Equations |
| Lesson 19: | Solving Equations Involving Grouping Symbols |
| Lesson 20: | Equations Involving an Unknown on Both Sides |
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| Lesson 22: | Changing The Subject of Simple Equations |
| Lesson 23: | Transposing Formulae |
| Lesson 24: | Solving Word Problems |

TOPIC 4: EQUATIONS



Welcome to Topic 4 of Unit 2 of Grade 9 Mathematics.

Yes, sentences are not only used in English, but also in Mathematics! In Algebra a sentence contains numbers, variables, operations and either an equal sign ($=$) or an inequality symbol.

Take a look at the following mathematical symbols for their corresponding meanings:

Symbol	Meaning
$=$	Equal to
$\neq$	Not equal to
$<$	Less than
$\leq$	Less than or equal to
$>$	Greater than
$\geq$	Greater than or equal to

A sentence that contains an equal sign ($=$) is called an **equation**.

Equations play a crucial role in modern mathematics and form the basis for mathematical modelling of numerous phenomena and processes in the field of science and engineering as well as in almost every fields of human endeavour.

In this topic, you will:

- revise the following terms: equation, members of an equation, identical equations, conditional equations, root of an equation, equation of the first degree in one unknown and solution of an equation.
- realize and appreciate the importance of solving the equation.
- Increase your skill, by the use of the properties of equations in solving an equation

Lesson 18: Simple Equations



You learnt something about equations in Grade 7 and 8 Mathematics.



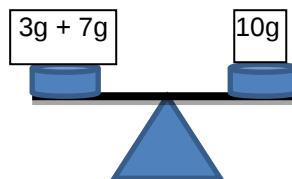
In this lesson, you will:

- define equation and identify its parts
- identify the properties and steps for solving equations
- solve equations

Definition of an Equation

An equation is a mathematical sentence or statement that two expressions are equal. The expressions are referred to as the members or sides of the equation.

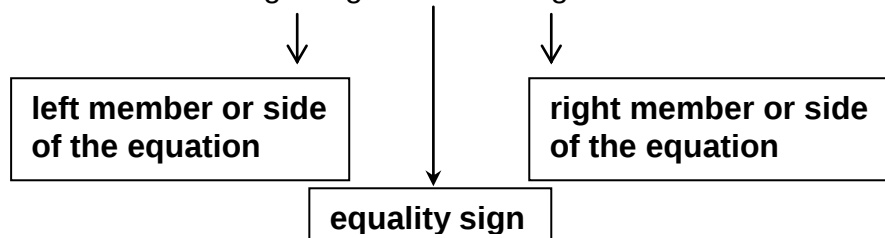
Consider the weighing scale shown below:



If the pans balance, what can be said about the weights in the two pans? What mathematical sentence describes the position of the two pans?

Since the pans balance, the weight on the left hand pan are together **equal** to the weight on the right hand pan. The mathematical sentence **$3g + 7g = 10g$** is an **equation**. It shows that the two expressions name the same number, are equivalent or have equal values.

The mathematical sentence: $3g + 7g = 10g$



An equation has a left side and a right side which are equal. If the value of the expression on the left side is the same as the value on the right side, then that is a **true equation**. If the values are different then the equation is **false**.

Examples

$2 + 5 = 8$ is a false equation

$3 + 6 = 9$ is a true equation

Equations or sentences that contain one or more variables that may be true or false depending on what values you substitute for the variables are called **open sentences or equations**.

There are two kinds of equations; namely:

1. **Identical equation or identity** is an equation whose members or sides are equal for all values substituted for the unknown.

Example 1

The equation $2(x - 4) = 2x - 8$ is an identity or identical equation since it is satisfied by any values for x .

2. **Conditional equation** is an equation whose members or sides are equal for certain values of the unknown or unknowns only.

Example 2

The equation $4x - 3 = 5$ is conditional equation, since it is satisfied only by $x = 2$

Equations containing only one variable can take any number which, when substituted for the variable makes the member of the equation equal to each other. Such a number is called the **root or solution** of the equation, and is said to **satisfy** the equation.

Consider the equation $x - 2 = 4$.

If we substitute **4** in place of x , our equation is false because $4 - 2 = 2$.

If we substitute **6** in place of x , our equation is true, because $6 - 2 = 4$.

Note that the only value for x that will make the equation true is 6. We call 6 as the **root or solution** of the equation.

Root or solution of the equation is the number that makes an equation true.

Here are some examples of an open sentence equation:

Example 1

a) $x + 1 = 5$

b) $3x = 6$

c) $x - 1 = 3$

Suppose we take values of the variable only from a specified set, say **(1, 2, 3, 4,...20)**, then we will call this set the **domain** of the variable.

A **domain** is the set of numbers that the variable may represent.
The domain is called a **replacement set**.

A **solution set** is the set of numbers that consists of the members of the domain of the variable for which the sentence is true.

Example 2

Solve each equation if the domain of x is $\{0, 1, 2, 3, 4, 5, 6\}$. If there is no solution in the given domain, state “no solution”.

Solution :

To solve the equations in example 1 , **substitute the values from the domain**, one at a time, and note which value will make the statement true.

a) $x + 1 = 5$

b) $3x = 6$

c) $x - 1 = 3$

Answer : 4

Answer: 2

Answer: 4

Properties to Solving Equations

In the previous lesson, we solved equations by substituting the members of the domain, one at a time, and noting which of the resulting equations were true. When the domain has only a few members, this can be difficult or even impossible.

We can use the following property to help solve certain types of equations.

Addition Property of Equality

Let a , b , and c be any real number.

If $a = b$, then $a + c = b + c$

When using the addition property of equality, we can say that we add or subtract both sides of the equation by the same number .

Example 3

To solve an equation like $x - 1 = 5$, we need to **isolate** the variable on one side of the equation.

Solution :

STEPS

$x - 1 = 5$

Write the equation

$x - \cancel{1} + \cancel{1} = 5 + 1$

Using the addition property

Add 1 to “get rid “ of -1 on the left hand side of the equation. Since -1 is the opposite of 1 . ($1 + -1 = 0$) Also, add 1 on the right hand side of the equation to isolate the variable.

$x = 6$

Solve for the variable

Check: $x - 1 = 5$
 $6 - 1 = 5$
 $5 = 5$

Substituting 6 for x
 The statement is TRUE so the solution $x = 6$ is correct

The solution is 6.

Example 4

To solve an equation like $x + 20 = 30$, we need to isolate the variable on one side of the equation.

Solution

$$x + 20 = 30$$

Write the equation

$$x + 20 + (-20) = 30 + (-20)$$

Using the addition property

Add (-20) to “get rid “ of 20 on the left hand side of the equation. Since -20 is the opposite of 20 . ($20 + -20 = 0$) Also, add -20 on the right hand side of the equation to isolate the variable.

$$x = 10$$

Solve for the variable

Check: $x + 20 = 30$

$$10 + 20 = 30$$

Substituting 6 for x

$$30 = 30$$

The solution is 10.

Multiplication Property of Equality

Let a, b, and c be any real number.

If $a = b$, then $ca = cb$

When using the multiplication property of equality, we can say that we multiply both sides of the equation by the same number.

Example 5

To solve an equation like $3y = 6$ we need to isolate the variable on one side of the equal sign.

Solution

STEPS

$$3y = 6$$

We must multiply by $\frac{1}{3}$, the reciprocal of 3.

$$3y \left(\frac{1}{3} \right) = 6 \left(\frac{1}{3} \right)$$

Using the multiplication Property

Multiply $3y$ by the reciprocal $\left(\frac{1}{3} \right)$ to get rid of 3.

Therefore, if we multiply both sides of the equation by $\left(\frac{1}{3} \right)$, we will be able to isolate the variable.

$$y = 2$$

Solve for the variable

Check: $3y = 6$

$$3 \times 2 = 6$$

$$6 = 6$$

Substituting y by 2

The solution is 2.

Example 6

Solve $\frac{2}{3}m = 18$

Solution:

$$\frac{2}{3}m \left(\frac{3}{2}\right) = 18\left(\frac{3}{2}\right)$$

Using the Multiplication Property

Multiply both sides by $\frac{3}{2}$ the reciprocal of $\frac{2}{3}$

$$m = 27$$

Solve for the variable

Check: $\frac{2}{3}m = 18$

$$\frac{2}{3}(27) = 18$$

$$\frac{54}{3} = 18$$

$$18 = 18$$

The solution is m = 27.

Example 7

Solve $\frac{w}{-5} = 7$

Solution:

$$\frac{w}{-5} = 7$$

Multiply by -5 (the opposite of $\div 5$) on both sides of the equation and simplify.

$$\frac{w}{-5} (-5) = 7(-5)$$

$$w = -35$$

Solve for the variable

The solution is w = -35.

Now look at these other examples. This time the equations contain like terms.

Example 8

Solve $2x + 4 + 5x - 8 = 24$

Solution:

$$2x + 4 + 5x - 8 = 24$$

$$7x - 4 = 24$$

$$7x - 4 + 4 = 24 + 4$$

$$7x = 28$$

Hence, $x = 4$

Collect like terms

Add 4 to both sides

Divide both sides by 7

To check: Substitute 4 for x.

$$2(4) + 4 + 5(4) - 8 = 24$$

$$8 + 4 + 20 - 8 = 24$$

$$32 - 8 = 24$$

$$24 = 24$$

Example 9

Solve $4h + 8 + 7h - 2h = 89$

Solution:

$$4h + 8 + 7h - 2h = 89$$

$$9h + 8 = 89$$

$$9h + 8 - 8 = 89 - 8$$

$$9h = 81$$

Hence, $h = 9$

Collect like terms

Subtract 8 to both sides

Divide both sides by 9

To check: Substitute 9 for h.

$$4(9) + 8 + 7(9) - 2(9) = 89$$

$$36 + 8 + 63 - 18 = 89$$

$$107 - 18 = 89$$

$$89 = 89$$

NOW DO PRACTICE EXERCISE 18

**Practice Exercise 18**

1. Write an equation for each of the following. Use the pronumeral in brackets to represent the unknown number.

Example: Seven is added to a number to give a result of ten (a)

Answer: $a + 7 = 10$

- (a) Four is subtracted from a number to give a result of sixteen (m)
 - (b) Nine times a number gives a result of sixty three (d)
 - (c) A number divided by seven is four (n)
 - (d) The sum of a number and eleven is fifteen (c)
 - (e) The sum of eleven and a number is zero (p)
-

2. Write the root or solution to each of the equation you found in Question 1.

- (a)
 - (b)
 - (c)
 - (d)
 - (e)
-

3. Which equation describes the sentence: "Five is added to a number to give a result of twelve"

- (a) $5 + 12 = n$
 - (b) $n + 12 = 5$
 - (c) $n + 5 = 12$
 - (d) $5n = 12$
-

4. Which of the equation describes this sentence: "Five is added to a three times a number to give a result of twenty".

- (a) $3p + 20 = 5$
- (b) $5 + 20 = 3p$
- (c) $5p + 3 = 20$
- (d) $3p + 5 = 20$

5. Determine whether the value given in the bracket makes the equation true.

(a) $m + 2 = 9$ ($m = 7$)

(b) $10 - p = 4$ ($p = 14$)

(c) $4(t + 5) = 16$ ($t = 1$)

(d) $4(5 - v) = 12$ ($v = 8$)

(e) $\frac{6 - x}{7} = 4$ ($x = 34$)

6. Solve the following equations.

(a) $x + 2 = 5$

(b) $u - 7 = 5$

(c) $6m = 48$

(d) $\frac{d}{4} = -5$

(e) $2p + 3 = -21$

(f) $1 + x - 5x + 7 = 0$

(g) $6x + 8 - x + 2 - 2 + x = 56$

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 4.
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Lesson 19: Solving Equations Involving Grouping Symbols



In Lesson 18, you revised the meaning of an equation and its parts. You also learnt to solve simple equations.



In this lesson, you will:

- identify the steps in solving equations with grouping symbols
- solve equations involving grouping symbols

Earlier in your study of Topic 3, you learnt how to simplify algebraic expressions by removing grouping symbols. Now you will be dealing with solving equations involving grouping symbols.

First let us revise what grouping symbols are.

Grouping symbols are symbols that are used to group numbers and variables. They indicate that the operation within them must be solved first before moving on to the other operations.

There are basically three types of grouping symbols: **parentheses ()**, **brackets []** and **braces { }**.

Now, imagine you have an equation that has variables on both sides like the one below.

$$4x = 2(x + 6)$$

This equation can be solved using the properties of equality we already know.

Our aim is to get a single x on its own.

First, we apply the distributive property to simplify the right side of the equation:

$$4x = 2(x + 6)$$

$$4x = 2x + 12$$

Next, we subtract $2x$ from each side: $2x = 12$

Next, we divide each side by 2: $x = 6$

The above discussion suggests the following:

To solve an equation containing brackets, we may proceed as follows:

- Remove the brackets by expanding according to the Distributive Law.
- Collect the variable terms on the left-hand side of the equation and the numerical terms on the right-hand side of the equation by using the same operation(s) on both sides of the equation.
- Then solve the equation by applying the same operation to both sides of it.

Here are various examples for you to understand how to solve equations involving grouping symbols.

Example 1

Solve $4(p+1) = 36$

Solution:

$$4(p + 1) = 36 \quad \{\text{Use Distributive Law}\}$$

$$4p + 4 = 36 \quad \{\text{Subtract 4 from both sides}\}$$

$$4p + 4 - 4 = 36 - 4$$

$$4p = 32 \quad \{\text{Divide both sides by 4}\}$$

$$\frac{4p}{4} = \frac{32}{4}$$

$$p = 8$$

The solution is **$p = 8$** .

To check: Substitute the value of p in the original equation.

$$4(p + 1) = 36$$

$$4(8 + 1) = 36$$

$$4(9) = 36 \longrightarrow 36 = 36 \text{ is a TRUE statement}$$

so $p = 8$ is a correct solution.

Example 2

Solve $14 = 28 + 7(1 - c)$

Solution:

$$14 = 28 + 7(1 - c) \quad \{\text{Use Distributive Law}\}$$

$$14 = 28 + 7 - 7c$$

$$14 = 35 - 7c \quad \{\text{Subtract 14 from both sides}\}$$

$$14 - 14 = 35 - 7c - 14$$

$$0 = 21 - 7c \quad \{\text{Add } 7c \text{ to both sides}\}$$

$$0 + 7c = 21 - 7c + 7c$$

$$7c = 21 \quad \{\text{Divide both sides by 7}\}$$

$$\frac{7c}{7} = \frac{21}{7}$$

$$c = 3$$

The solution is **$c = 3$** .

To check: Substitute the value of c in the original equation.

$$14 = 28 + 7(1 - c)$$

$$14 = 28 + 7(1 - 3)$$

$$14 = 28 + 7(-2)$$

$$14 = 28 - 14 \longrightarrow 14 = 14 \text{ is a TRUE statement}$$

so $c = 3$ is a correct solution.

Some grouping symbols are preceded by a negative sign. In problems like this, you still have to use the distributive law, just do not forget to distribute the negative sign as well.

Example 3

Solve $12 - (-b - 6) = 20$.

Solution:

$$\begin{aligned}
 12 - (-b - 6) &= 20 && \{\text{Use Distributive Law}\} \\
 12 + b + 6 &= 20 && \{\text{Remember to distribute the negative sign}\} \\
 18 + b &= 20 \\
 18 - 18 + b &= 20 - 18 && \{\text{Subtract 18 on both sides}\} \\
 b &= 2
 \end{aligned}$$

To check: Substitute the value of b in the original equation.

$$\begin{aligned}
 12 - (-b - 6) &= 20 \\
 12 - (-2 - 6) &= 20 \\
 12 - (-8) &= 20 \longrightarrow 10 = 10 \text{ is a TRUE statement} \\
 \text{so } b = 2 &\text{ is a correct solution.}
 \end{aligned}$$

Example 4

Solve $-4(d - 6) = 0$.

$$\begin{aligned}
 -4(d - 6) &= 0 \\
 -4d + 24 &= 0 && \{\text{Use Distributive Law}\} \\
 -4d + 24 - 24 &= 0 - 24 && \{\text{Remember to distribute the negative sign}\} \\
 -4d &= -24 \\
 -4(-1/4)d &= -24(-1/4) && \{\text{Get rid of -4 by multiplying both sides by } (-1/4)\} \\
 d &= 6
 \end{aligned}$$

To check:

$$\begin{aligned}
 -4(d - 6) &= 0 \\
 -4(6 - 6) &= 0 \\
 -4(0) &= 0 \\
 0 &= 0
 \end{aligned}$$

Now look at the following examples on the next page.

Example 4 Solve the equation $6(x - 2) = 3 - 5(x + 14)$.

These equations are multistep equations with grouping symbol on both sides.

Solution:

$$6(x - 2) = 3 - 5(x + 14)$$

$$6x - 12 = 3 - 5x - 70$$

Distributive property

$$6x - 12 = -67 - 5x$$

Collect like terms

$$6x + 5x - 12 = -67 - 5x + 5x$$

Add 5x to both sides

$$11x - 12 = 3 - 70$$

$$11x - 12 + 12 = 3 - 70 + 12$$

Add 12 to both sides

Hence, $11x = -55$

Divide both sides by 11

$$x = -5$$

To check: Substitute -5 for x in the original equation.

$$6(-5 - 2) = 3 - 5(-5 + 14)$$

$$6(-7) = 3 - 5(9)$$

$$-42 = 3 - 45$$

Example 5 Solve $x + (3x - 7) = 4x - (x + 2)$

Solution:

$$x + (3x - 7) = 4x - (x + 2)$$

$$x + 3x - 7 = 4x - x + 2$$

Distributive property

$$4x - 7 = 3x - 2$$

Collect like terms

$$4x - 3x - 7 + 7 = 3x - 2 - 3x + 7$$

Subtract 3x from both sides and add 7 on both sides

$$x = 5$$

Check: Substitute 5 for x in the original equation.

$$5 + [(3)(5) - 7] = 4(5) - (5 + 2)$$

$$5 + (15 - 7) = 20 - 7$$

$$5 + 8 = 13$$

$$13 = 13$$

NOW DO PRACTICE EXERCISE 19

**Practice Exercise 19**

Solve each of the following equations and check your solution by substitution:

1. $2(t + 1) = 4$

2. $-4(j + 2) = 16$

3. $7(3 - v) = -14$

4. $-6(8 + f) = 6$

5. $9(2c - 8) = 0$

6. $(k - 2) - 9 = 11$

7. $6x - 2(x + 3) = 5(2x - 1)$

8. $3x + 2 - 4(x - 3) = 2(5x - 4)$

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 4

Lesson 20: Equations Involving Unknown on Both Sides



You learnt to solve equations involving grouping symbols in the last lesson.



In this lesson, you will:

- identify the steps in solving equations with the unknown on both sides
- solve equations with the unknown on both sides.

Some problems produce equations that have variables on both sides of the equal sign.

For instance, Sera runs the same distance each day. On Mondays, Fridays and Saturdays, she runs 3 laps on the track and an additional 5 m off the track. On Tuesdays and Thursdays, she runs 4 laps and 2.5 m off the track.

Expression for
Mondays, Fridays
and Saturdays

$$3x + 5$$

$$4x + 2.5$$

Expression for
Tuesday and
Thursday

$$3x + 5 = 4x + 2.5$$

The variable x in this expression is the length of one lap of the track. Since the total distance each day is the same, the two expressions are equal.

Solving an equation with variables on both sides is similar to solving an equation with a variable on one side only. You can add or subtract a term containing a variable on both sides of an equation.

Example 1

Solve $3a = 2a + 3$

Solution: $3a = 2a + 3$

$$3a - 2a = 2a + 3 - 2a \quad \text{subtract } 2a \text{ from both sides}$$

$$a = 3$$

Check: Substitute 3 for a on the original equation.

$$3(3) = 2(3) + 3$$

$$9 = 6 + 3$$

$$9 = 9$$

Example 2

Solve $5 + 7v = 4v - 7$

Solution:	$5 + 7v = 4v - 7$	
	$5 + 7v - 4v = 4v - 7 - 4v$	subtract $4v$ from both sides
	$5 + 3v = -7$	collect like terms
	$5 - 5 + 3v = -7 - 5$	subtract 5 from both sides
	$3v = -12$	simplify
	$\frac{3v}{3} = \frac{-12}{3}$	divide both sides by 3
	$v = -4$	

Check: Substitute -4 for v on the original equation.

$$\begin{aligned}
 5 + 7(-4) &= 4(-4) - 7 \\
 5 - 28 &= -16 - 7 \\
 -23 &= -23
 \end{aligned}$$

The key to solving these types of equations is to move all the terms containing the variable to one, and only one, side. It doesn't matter which side you choose, just try to pick the easier one.

Here are other examples.

Example 3

Solve $2x - 6 = 5x + 18$

Solution:	$2x - 6 = 5x + 18$	
	$2x - 6 - 2x = 5x + 18 - 2x$	subtract $2x$ from both sides
	$-6 = 3x + 18$	collect like terms
	$-6 - 18 = 3x + 18 - 18$	subtract 18 from both sides
	$-24 = 3x$	simplify
	$\frac{-24}{3} = \frac{3x}{3}$	divide both sides by 3
	$-8 = x$	

Check: Substitute -8 for x in the original equation..

$$\begin{aligned}
 2(-8) - 6 &= 5(-8) + 18 \\
 -16 - 6 &= -40 + 18 \\
 -22 &= -22
 \end{aligned}$$

Example 4

Solve $5x + 2 = 3x - 5$

Solution: $5x + 2 = 3x - 5$

$$5x + 2 - 3x = 3x - 5 - 3x$$

Subtract $3x$ from both sides

$$2x + 2 = -5$$

collect like terms

$$2x + 2 - 2 = -5 - 2$$

Subtract 2 from both sides

$$2x = -7$$

simplify

$$\frac{2x}{2} = \frac{-7}{2}$$

Divide both sides by 2

$$x = \frac{-7}{2}$$

Answer

Remember:

To solve equations with variables on both sides follow the following steps:

- 1. Combine or collect like terms and clear fractions if there is any by multiplying all terms by the denominator.**
- 2. Use the Distributive Property if needed.**
- 3. Add or subtract variable terms to both sides so that the variable occurs only on one side of the equation. And the constants on the other side.**
- 4. Use properties of equality to isolate the variable.**
- 5. Check the equation**

Now look at Example 5.

Check by substitution, if the given value for x given in the parentheses is the correct solution to the following equations.

(a) $2x - 5 = 10 - 3x$ **($x = 3$)**

The given value for x is 3, now you have to substitute $x = 3$ into the given equation to check if $x=3$ is correct.

$$2x - 5 = 10 - 3x$$

$$2(3) - 5 = 10 - 3(3)$$

$$6 - 5 = 10 - 9$$

$$1 = 1$$

Since $LHS = RHS$, then the given answer $x = 3$ is a correct solution.

(b) $5x + 2 = 2x - 7$ **($x = 2$)**

Substitute $x = 2$ into the given equation.

$$5x + 2 = 2x - 7$$

$$5(2) + 2 = 2(2) - 7$$

$$10 + 2 = 4 - 7$$

$$12 \neq -3$$

The right hand side of the equation does NOT equal the left hand side, therefore the answer $x = 2$ is NOT the solution.

NOW DO PRACTICE EXERCISE 20

**Practice Exercise 20**

1. Solve the following equations.

(a) $8x + 7 = 4x - 2$

(b) $7x + 3 = 2x + 7$

(c) $5 + 2x = 11 - x$

(d) $x - 3 = 5x + 7$

2. Check to see if the following solution for each equation is correct.

a. $3 - x = -12 + 4x$ ($x = 5$)

b. $15 - 3a = 10 - a$ ($a = 1$)

3. Solve the following equations.

(a) $3(x - 1) = 2x + 9$

(b) $5x + 1 = -(x + 3)$

(c) $3y + 2 = 5(y - 6)$

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 4

Lesson 21: Equations Involving Fractions



You learnt in the last lesson how to identify the steps in solving equations with variable on both sides and use them to solve the equations.



In this lesson, you will:

- identify the rules and steps in solving equations involving fractions
- solve equations involving fractions.

In algebra, while evaluating the value of the unknown, the coefficient of the unknown may have a fraction.

You can solve an equation containing fractions by following the steps below:

1. Find the least common denominator (LCD) of all the fractions in the equation.
2. Remove fractions by multiplying both sides of the equation by the LCD.
3. Solve the equation for the unknown or variable by performing the same operations to both sides of the equation.
4. Check the equation.

The LCD of the fractions is the least common multiple (LCM) of the denominators of the given fractions.

Now look at the examples.

Example 1

Solve: $\frac{x}{3} = \frac{2}{5}$

Solution: The LCM of 3 and 5 is 15. So the LCD is 15.

$$15\left(\frac{x}{3}\right) = 15\left(\frac{2}{5}\right) \quad \text{Multiply both sides by 15}$$

$$5x = 6 \quad \text{Simplify}$$

$$\frac{5x}{5} = \frac{6}{5} \quad \text{Divide both sides by 5}$$

$$x = \frac{6}{5} \text{ or } 1\frac{1}{5} \quad \text{Answer}$$

Check: Substitute $\frac{6}{5}$ for x in the original equation.

$$\frac{\frac{6}{5}}{3} = \frac{2}{5}$$

$$\frac{6}{5} \left(\frac{1}{3} \right) = \frac{2}{5}$$

$$\frac{2}{5} = \frac{2}{5}$$

Example 2

Solve: $\frac{2x}{5} + 1 = \frac{13}{5}$

Solution: Since 5 is the only denominator, 5 is the LCD.

So, $5 \left(\frac{2x}{5} + 1 = \frac{13}{5} \right) 5$ Multiply both sides by 5

$\cancel{5} \left(\frac{2x}{\cancel{5}} + 1 = \frac{13}{\cancel{5}} \right) \cancel{5}$ Cancel the 5's on the LHS and RHS of the equation

$2x + 5 = 13$ Simplify

$2x + 5 - 5 = 13 - 5$ Subtract 5 from both sides

$2x = 8$ Collect like terms

$\frac{2x}{2} = \frac{8}{2}$ Divide both sides by 2

$x = 4$ Answer

Check: Substitute 4 for x in the original equation.

$$\frac{2(4)}{5} + 1 = \frac{13}{5}$$

$$\frac{8}{5} + 1 = \frac{13}{5}$$

$$\frac{8+5}{5} = \frac{13}{5}$$

$$\frac{13}{5} = \frac{13}{5}$$

Example 3 Solve: $\frac{x}{7} + \frac{x}{3} = 10$

Solution: The LCD of the fractions is 21.

$$21\left(\frac{x}{7} + \frac{x}{3}\right) = 21(10) \quad \text{Multiply both sides by 21}$$

$$3x + 7x = 210 \quad \text{Simplify}$$

$$10x = 210 \quad \text{Collect like terms}$$

$$\frac{10x}{10} = \frac{210}{10} \quad \text{Divide both sides by 10}$$

$$x = 21 \quad \text{Answer}$$

Check: Substitute 21 for x in the original equation

$$\frac{21}{7} + \frac{21}{3} = 10$$

$$3 + 7 = 10$$

$$10 = 10$$

Example 4 Solve $\frac{x+2}{8} = \frac{2}{3}$ for x.

Solution: $\frac{x+2}{8} = \frac{2}{3}$ LCM of 8 and 3 is 24. So, The LCD is 24.

$$24\left(\frac{x+2}{8}\right) = (24)\frac{2}{3} \quad \text{Multiply both sides by 24}$$

$$3(x+2) = 16 \quad \text{Simplify}$$

$$3x + 6 = 16 \quad \text{Remove brackets}$$

$$3x + 6 - 6 = 16 - 6 \quad \text{Subtract 6 from both sides}$$

$$3x = 10 \quad \text{Collect like terms}$$

$$\frac{3x}{3} = \frac{10}{3} \quad \text{Divide both sides by 3}$$

$$x = \frac{10}{3} \text{ or } 3\frac{1}{3} \quad \text{Answer}$$

Check: $\frac{\frac{10}{3} + 2}{8} = \frac{2}{3}$

$$\frac{\frac{10+6}{3}}{8} = \frac{2}{3}$$

$$\frac{\frac{16}{3}}{8} = \frac{2}{3}$$

$$\frac{\frac{2}{3}}{8} = \frac{2}{3}$$

Here is another example

Example 5 Solve the equation $\frac{x}{2} - \frac{3x}{4} + \frac{1}{3} = x + \frac{7}{6}$

Solution: $\frac{x}{2} - \frac{3x}{4} + \frac{1}{3} = x + \frac{7}{6}$

LCD is 12.

$$12\left(\frac{x}{2} - \frac{3x}{4} + \frac{1}{3}\right) = 12\left(x + \frac{7}{6}\right)$$

Multiply both sides by LCD, 12

$$12\left(\frac{x}{2}\right) - 12\left(\frac{3x}{4}\right) + 12\left(\frac{1}{3}\right) = 12(x) + 12\left(\frac{7}{6}\right)$$

Distributive property

$$6x - 9x + 4 = 12x + 14$$

Simplify

$$-3x + 4 = 12x + 14$$

Collect like terms

$$-3x - 12x + 4 = 12x - 12x + 14$$

Subtract 12x on both sides

$$-15x + 4 = 14$$

$$-15x + 4 - 4 = 14 - 4$$

Subtract 4 on both sides

$$-15x = 10$$

Divide both sides by 15

$$x = -\frac{2}{3}$$

Answer

Check: Substitute $-\frac{2}{3}$ for x in the original equation.

$$\frac{-\frac{2}{3}}{2} - \frac{3\left(-\frac{2}{3}\right)}{4} + \frac{1}{3} = -\frac{2}{3} + \frac{7}{6}$$

$$-\frac{2}{3}\left(\frac{1}{2}\right) - \frac{-2}{4} + \frac{1}{3} = -\frac{2}{3} + \frac{7}{6}$$

$$-\frac{1}{3} + \left(\frac{1}{2}\right) + \frac{1}{3} = -\frac{2}{3} + \frac{7}{6}$$

$$-\frac{1}{3} + \frac{1}{2} + \frac{1}{3} = -\frac{2}{3} + \frac{7}{6}$$

$$\frac{1}{2} = \frac{3}{6}$$

$$\frac{1}{2} = \frac{1}{2}$$

NOW DO PRACTICE EXERCISE 21

**Practice Exercise 21**

1. Solve for x:

(a) $\frac{x}{3} = 5$

(b) $\frac{1 + 2x}{7} = 6$

2. Solve: $\frac{1 - x}{2} = \frac{x + 2}{3}$

3. Solve for x:

(a) $\frac{x}{3} = \frac{2}{5}$

(b) $\frac{x}{2} = 4$

(c) $\frac{4}{7} = \frac{x}{3}$

4. Solve for x

(a) $\frac{x-1}{2} = 6$

(b) $\frac{1}{4}(5-2x) = -2$

(c) $\frac{x+5}{3} = -1$

5. Solve for x:

(a) $\frac{2x+1}{3} = \frac{1}{2}$

(b) $\frac{3x+2}{5} = \frac{x-1}{4}$

(c) $\frac{2x-7}{3} = x-5$

(d) $\frac{2x-1}{7} = \frac{3x}{5}$

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 4

Lesson 22: Changing Subjects of Simple Equations



You learnt to identify and apply the steps on how to solve equations involving fractions in the last lesson.



In this lesson, you will:

- use the steps in solving equations to change the subject of an equation.

Thus far the equations we have solved contained only one variable. Now, you will look at equations which involve two or more variables. These are called **literal equations** (or **letter equations**).

Literal equation is an equation which involves two or more variables.

- Examples:
- (a) $ay = 5p - by$
 - (b) $f = v + w$
 - (c) $d = rt$
 - (d) $ax + ay = az$

One of the very powerful things that Algebra can do is to "rearrange" the equation so that another variable is the subject. We call this process **changing the subject of an equation or formula**.

The objective or aim of **changing the subject** of an **equation** is to get the **variable** you want on one side of the '=' sign by itself, and **positive** if possible. The process is very similar to solving equations, as the following examples demonstrate.

Below are some examples.

- Make x the subject of $a + x = b$
 $a - a + x = b - a$ Subtract a from both sides
 $x = b - a$

In solving the equation we have made x the subject of the formula.

- Make x the subject of $k - x + m = 0$
 $k + m - x + x = 0 + x$ Add x to both sides
 So, $k + m = x$ or $x = k + m$

As you can see, changing the subject (also referred to as **rearranging**) works with numbers or variables. An examination question may use the phrase '**in terms of x** ' - this just means that **x is to be the subject of your rearrangement**. So, the following all mean the same:

- Rearrange $y + 2x = 0$ in terms of x
- Make x the subject of $y + 2x = 0$
- Change the subject of $y + 2x = 0$ to x

Sometimes, there may be several steps involved in the rearrangement. Just remember to make one change at a time, each change being aimed at putting the chosen variable on one side and all other variables on the other.

Examples

1. Make x the subject of $3x + 5 = 2y$

$$3x = 2y - 5$$

Subtract 5 from both sides

$$x = \frac{2y - 5}{3}$$

Divide both sides by 3
2. Make x the subject of $vx - n = e$

$$vx = e + n$$

Add n to both sides

$$x = \frac{e + n}{v}$$

Divide both sides by v
3. Make x the subject of $v + 2x = w + y$

$$2x = w + y - v$$

Subtract v from both sides

$$x = \frac{w + y - v}{2}$$

Divide both sides by 2

Changing the subject of equations with brackets

If an equation contains brackets, you must **expand** the brackets before rearranging the equation..

Examples

1. Make x the subject of $a(x + 4) = y$

$$ax + 4a = y$$

Multiply out the brackets

$$ax = y - 4a$$

Subtract 4a from each side

$$x = \frac{y - 4a}{a}$$

Divide both sides by a, giving
2. Make x the subject of $a(x - b) = 3c$

$$ax - ab = 3c$$

Multiply out the brackets

$$ax = 3c + ab$$

Add ab to both sides

$$x = \frac{3c + ab}{a}$$

Divide both sides by a
3. Make x the subject of $c(a + x) = d$.

$$ca + cx = d$$

Multiply out the bracket

$$cx = d - ca$$

Subtract ca from each side

$$x = \frac{d - ca}{c}$$

Divide both sides by c

4. Make x the subject of $x(a + 4) = y$

As x is outside the brackets, you don't need to expand them. Just divide both sides by $(a + 4)$, giving

$$\frac{x(a + 4)}{(a + 4)} = \frac{y}{a + 4}$$

$$x = \frac{y}{a + 4}$$

Changing the subject of an equation containing a fraction

If an equation contains a **fraction**, the equation will need to be rearranged, as the following examples demonstrate.

Examples

1. Make x the subject of $y = \frac{x}{4}$.

$$x = 4y$$

Eliminate the fraction by multiplying both sides by 4.

2. Make x the subject of $y = \frac{x}{2} - 5$.

$$y + 5 = \frac{x}{2}$$

Add 5 to both sides

$$2(y + 5) = 2\left(\frac{x}{2}\right)$$

Multiply both sides by 2

$$x = 2y + 10$$

3. Make x the subject of $\frac{x}{h} = \frac{c}{d}$.

$$x = \frac{ch}{d}$$

Multiply both sides by h

4. Make x the subject of $\frac{a}{b} = \frac{c}{x}$.

$$ax = bc$$

Multiply both sides by x

$$ax = bc$$

Multiply both sides by b

$$x = \frac{bc}{a}$$

Divide both sides by a

NOW DO PRACTICE EXERCISE 22

**Practice Exercise 22**

1. Make x the subject of the following equations.

(a) $y = x + 3$

(g) $y = mx + c$

(b) $x + m = y$

(h) $y = 3x - 7$

(c) $y = x - 5$

(i) $y = \frac{x}{2} + 5$

(d) $y = x - m$

(j) $y = \frac{3x}{4} + 5$

(e) $y = 8x$

(k) $y = \frac{xw}{t} + p$

(f) $y = 2x + 5$

(l) $y = \frac{5x + 2}{7}$

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 4

Lesson 23: Transposing Formulae



You learnt to change the subject of an equation by using the steps of solving equations in the last lesson.



In this lesson, you will:

- define a formula
- transpose formulas

Earlier we defined an equation is a mathematical sentence or statement that two expressions or quantities are equal.

An equation says that two things are equal. It will have an equals sign "=" like this:

$$x + 2 = 6$$

That equation says: **what is on the left ($x + 2$) is equal to what is on the right (6).**

So an equation is like a statement "**this equals that**".

We also defined a literal equation as an equation which involves two or more variables.

Now we will look at a special type of literal equation called **formula**. Many familiar formulae such as the following:

$$A = lw \quad P = 2l + 2w \quad d = rt \quad A = \pi r^2 \quad I = prt$$

are literal equations.

A formula is an equation which specifies how a number of variables are related to one another.

Formulas are written so that a single variable, the **subject** of the formula, is on the left hand side of the equation. Everything else goes on the right hand side of the equation.

The "subject" of a formula is the single variable (usually on the left of the "=") that everything else is equal to.

Example 1 In the formula $d = rt$,
"d" is the subject of the formula.

Example 2 In the formula $s = ut + \frac{1}{2}at^2$
"s" is the subject of the formula

When working with formulas we sometimes substitute values for all variables except one and then solve to find the value of that variable. Other times, it is desirable to solve the formula for a particular variable without first substituting values for the other variables.

For example, the equation $d = rt$, might be solved for t ,

$$t = \frac{d}{r}$$

or for r ,

$$r = \frac{d}{t}.$$

We call this process as **changing the subject of the formula**.

To change the subject of the formula is to re-arrange the variables in the formula to make the required variable the subject. This is called **transposition**. The rules for transposing formulas are the same as those you have already used to solve equations.

Remember that the variables in a literal equation or a formula simply represent numbers. Any of the rules for solving equations involving numerical values are applicable when solving formulas.

Example 1

Make V the subject of the formula $V - E + F = 2$.

Solution:

$$V - E + F = 2$$

$$V = 2 + E - F$$

Add E and subtract F from both sides of the equation.

Example 2

Make E the subject of the formula $V - E + F = 2$

Solution:

$$V - E + F = 2$$

$$-E = 2 - V - F$$

$$E = -2 + V + F$$

Subtract V and F from both sides of the equation.

Multiply both sides by -1

Example 3

Make t the subject in: $v = u + at$

Solution:

$$v = u + at$$

$$v - u = at$$

$$\frac{v - u}{a} = t$$

$$t = \frac{v - u}{a}$$

Subtract u from both sides of the equation.

Divide both sides of the equation by a .

Here are some examples of formulas with squares and square roots

Example 4

Make b the subject of the formula $c^2 = a^2 + b^2$

Solution

$$\begin{aligned} c^2 &= a^2 + b^2 \\ c^2 - a^2 &= b^2 \\ b^2 &= c^2 - a^2 \\ b &= \sqrt{c^2 - a^2} \end{aligned}$$

Subtract a^2 from both sides of the equation

Get the square root of both sides of equation to find b.

Example 5

Find a formula for A from $r = \sqrt{\frac{A}{\pi}}$

Solution:

$$\begin{aligned} r &= \sqrt{\frac{A}{\pi}} \\ r^2 &= \frac{A}{\pi} \\ \pi r^2 &= A \\ A &= \pi r^2 \end{aligned}$$

Square both sides to undo the square root sign on the right.

Multiply π on both sides of the equation

Keep in mind that solving a formula depends on your ability to isolate the variable on one side of the equation.

When the variable you wish to solve occurs in more than one term, you collect these terms on one side of the equation or formula.

Example 6

Solve for y in the formula $ay = 5p - by$.

Solution:

$$\begin{aligned} ay &= 5p - by \\ ay + by &= 5p \\ y(a + b) &= 5p \\ y &= \frac{5p}{a + b} \end{aligned}$$

Add by to both sides of the equation.

Factor out y using the Distributive Law

Divide both sides by $a + b$

NOW DO PRACTICE EXERCISE 23

**Practice Exercise 23**

1. Change the subject of each formula to the pronumeral indicated in the brackets:

(a) $I = PRT$ (T)

(b) $V = u + at$ (a)

(c) $C = 2\pi r$ (r)

(d) $y = mx + b$ (x)

(e) $D = \frac{M}{V}$ (M)

2. Solve each of the following formulas for indicated variable.

(a) $A = LW$ for L .

(b) $C = \pi D$ for D .

(c) $V = \pi r^2 h$ for h .

(d) $A = \frac{1}{2}bh$ for b .

(e) $P = 2L + 2W$ for W .

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 4

Lesson 24: Solving Word Problems



You learnt how to solve several types of equations in the previous lessons.



In this lesson you will:

- translate word problems into equations
 - identify rules in solving worded problems
 - use equations to solve worded problems.
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Success with applied problems (or word problems) comes with practice and knowing some simple techniques of translation. Solving applied problems using algebra involves two steps. First, translate the words into algebraic equation. Second, solve the resulting equation.

Many problems which are stated in words have to be translated into mathematical symbols or algebraic expressions before they can be solved.

The following illustrative examples represent one number in terms of another.

Example 1

Nathan is four times as old as John. If x represents John's age, how would you represent Nathan's age?

Solution: John's age is x .

Since Nathan's age is four times John's age, four times x is $4x$.

Therefore, $4x$ represents Nathan's age.

Example 2

Manuel has 80 toea in his two pockets. If he has x toea in one pocket, how much has he in the other pocket?

Solution: Let x = amount in one pocket.

His total amount of money is 80 toea. Subtracting x toea, in the first pocket, from 80 toea gives us the amount in the other pocket.

Therefore, $80 - x$ is the amount in the other pocket.

It is necessary to be able to read word problems with understanding and to translate them into algebraic symbols correctly.

The following suggestions on the next page will help you to set up equations for solving word problems.

Here are procedures you have to follow in solving word problems.

1. Read the whole problem carefully and get a clear understanding of everything that is in the word problem.
2. Reread the problem and look for the following:
 - (a) What needs to be found
 - (b) All the relevant data
 - (c) Key words that suggest which operation to use
3. Represent the unknown quantity by a letter or a symbol.
4. Make a sketch or diagram whenever possible.
5. Form an equation expressed in symbols from the stated relations in the problem.
6. Solve the equation.
7. Check with the problem, not with the equation.

The following are certain keywords which indicate the mathematical operation you will use.

Key words for addition (+)	plus, increased by, more than, combined together, total of, sum, added to, increased by, more than, combined together, total of, sum, added to
Key words for Subtraction (–)	minus, less than, fewer than, reduced by, decreased by, difference of
Key words for multiplication (• x or integers next to each other 5y, xy)	of, times, multiplied by, the product of
Key words for division (÷, /)	per, a, out of, ratio of, quotient of, per cent (divide by 100)

Now look at the examples of solving word problems using equations.

Example 1

Twice a number increased by 10 equals 30. What is the number?

Solution: Let x represents the number.

Key words: $\underbrace{\text{Twice a number}}_{2x} \underbrace{\text{increased by 10}}_{+ 10} \underbrace{\text{equals 30}}_{= 30}$

The equation is written as $2x + 10 = 30$.

Solving the equation: $2x + 10 = 30$

$$2x = 20$$

$$x = 10$$

Therefore, the number is 10.

Check: $2(10) + 10 = 30$
 $20 + 10 = 30$
 $30 = 30$

Example 2

The sum of two consecutive even integers is 94. Find the numbers.

Solution: Even integers are whole numbers which are exactly divisible by 2. They differ by 2. (e.g. 2, 4, 6, 8, 10, ...24, 26,)

Let x = the first even integer

$x + 2$ = the second even integer

Key words: The $\underbrace{\text{sum of the two consecutive even integer}}_{x + x + 2}$ is $\underbrace{94}_{= 94}$

Form the equation: $x + x + 2 = 94$

Solve the equation: $x + x + 2 = 94$

$$2x + 2 = 94$$

$$2x = 92$$

$$x = 46 \quad (\text{first even integer})$$

$$x + 2 = 46 + 2$$

$$= 48 \quad (\text{second even integer})$$

Check: $46 + 48 = 94$
 $94 = 94$

Example 3

Find two consecutive odd integers whose sum is 44.

Solution: Odd integers are whole numbers which are not exactly divisible by 2. They also differ by 2. (e.g. 3, 5, 7, ..., 13, 15,...)

Let x = the first odd integer

$x + 2$ = the second integer

$x + x + 2$ = their sum

44 = the given sum

$x + x + 2 = 44$ (the equation)

Solve the equation: $x + x + 2 = 44$

$$2x + 2 = 44$$

$$2x = 42$$

$$x = 21 \quad (\text{first odd integer})$$

$$x + 2 = 21 + 2$$

$$= 23 \quad (\text{second odd integer})$$

Check: $21 + 23 = 44$
 $44 = 44$

Example 4

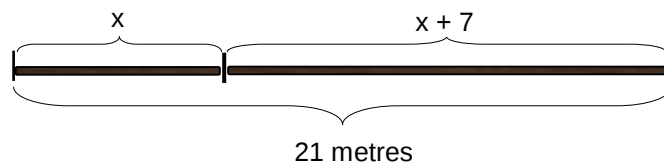
A 21-m rope is cut into two pieces, one 7m longer than the other. How long is each piece?

Solution: Let x = the length of the shorter piece

$x + 7$ = the length of the longer piece

21 m = total length

Make a sketch or diagram as shown below.



Translate the problem in to symbols to form the equation.

Hence, the equation is: $x + x + 7 = 21$

Solve the equation: $x + x + 7 = 21$

$$2x + 7 = 21$$

$$2x = 14$$

$$x = \frac{14}{2}$$

$$x = 7 \text{ m} \quad (\text{the length of the shorter piece})$$

and

$$x + 7 = 7 + 7$$

$$= 14 \text{ m} \quad (\text{the length of the longer piece})$$

The pieces are 7m and 14 m.

Check: $7 \text{ m} + 14 \text{ m} = 21 \text{ m}$

All the problems we have seen so far are number problems. Now let us solve other examples. This time we will look at other types of word problems.

Age Problems

To find the age of any person n years ago we subtract n from his/her age at present.
To find the his/her age n years from now we add n to his present age.

Suppose a man is x years old now, then $x - n$ is his age n years ago. Also $x + n$ will be his age n years from now or n years hence.

In solving age problems, it will be helpful to arrange the data of the problem in a tabulated form.

Example 5

Ten years ago Nicholas was twice as old as his sister Cassadee. In eight years the sum of their ages will be 63 years.

Find the present age of each.

Solution: Represent Cassadee's age ten years ago by x .

	Age 10 years ago	Present age	Age in 8 years
Nicholas	$2x$	$2x + 10$	$(2x + 10) + 8$
Cassadee	x	$x + 10$	$(x + 10) + 8$

Key words: In eight years the sum of their ages will be 63.

The equation is: $(2x + 10) + 8 + (x + 10) + 8 = 63$

Solve the equation: $2x + 10 + 8 + x + 10 + 8 = 63$

$$3x + 36 = 63$$

$$3x = 63 - 36$$

$$3x = 27$$

$$x = 9 \quad (\text{Cassadee's age 10 years ago})$$

Hence, Nicholas' age 10 years ago is $2x = 2(9)$

$$= 18 \text{ years}$$

Therefore, Cassadee's age at present is $x + 10 = 9 + 10 = \mathbf{19 \text{ years}}$.

Nicholas' age at present is $2x + 10 = 2(9) + 10 = 18 + 10 = \mathbf{28 \text{ years}}$

Also, Cassadee's age in 8 years is $19 + 8 = 27$

Nicholas' age in 8 years is $28 + 8 = 36$

Check: $36 + 27 = 63$

$$36 + 27 = 63$$

$$63 = 63$$

Many word problems require the use of certain basic formulas. In particular, problems involving geometric figures and rate problems might involve one of the formulas you have already learnt earlier.

As you read through each problem, try to determine which formula is appropriate and remember to make a sketch or diagram.

Now study the examples on the next pages.

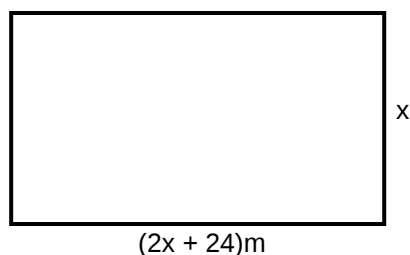
Example 5

A rectangular lot is 24 metres longer than twice its width. The perimeter of the lot is 210 metres.

What are the length and the width of the lot?

Solution: Recall that the perimeter of a rectangle is given by $P = 2L + 2W$.

It is helpful to make a sketch or diagram as shown below.



Representation: Let x = the width

$2x + 24$ = the length

210 = perimeter

Substitute these representations to $P = 2L + 2W$ to form your equation.

Hence, the Equation is: $210 = 2(x) + 2(2x + 24)$ or $2x + 2(2x + 24) = 210$

Solve the equation: $2x + 4x + 48 = 210$

$$6x = 210 - 48$$

$$6x = 162$$

$$x = 27 \text{ m} \quad (\text{width})$$

$$2x + 24 = 2(27) + 24$$

$$= 78 \text{ m} \quad (\text{length})$$

Therefore, the width is 27 m and the length is 78 m.

$$\text{Check: } 2(27) + 2(78) = 210$$

$$54 + 156 = 210$$

$$210 = 210$$

Another important formula relates the distance (D) that an object travels in a given time (t) at a constant or uniform rate (r).

$$\text{Distance} = \text{rate} \times \text{time} \quad \text{or} \quad D = rt$$

For example, a car traveling at a rate of 55 km/h for 3 hours will travel a distance

$$D = r t$$

$$= (55)(3)$$

$$= 165 \text{ km.}$$

All motion problems depend in some way in this formula.

Example 6

If a runner covers a distance of 96.6 metres in 9.2 seconds, how fast (at what rate) is he running?

Solution: Let r = his rate

9.2 sec = time he runs

96.6 m = distance he covers

We must solve the following equation:

$$D = rt$$

$$96.6 = r(9.2)$$

$$\frac{96.6}{9.2} = r$$

$$10.5 = r$$

Therefore, he is running **10.5 m/sec.**

NOW DO PRACTICE EXERCISE 24

**Practice Exercise 24**

1. Letting x represent the unknown number, translate the following into symbols.
 - (a) Seven times a number is 42.
 - (b) A number increased by 5 is 37.
 - (c) A number subtracted from 8 is 1.
 - (d) Twice a number, increased by 4, is 20.
 - (e) Tom's age in 3 years will be 39.

2. Solve each of the following word problems.
 - (a) The sum of two numbers is 52. The larger is three times the smaller number. Find the two numbers.

 - (b) The sum of two consecutive even integers is 170. Find the two integers

- (c) Sam is four times as old as Jim, and the difference of their ages is 33 years. How old is each?
- (d) The perimeter of a rectangle is 52 m. if the length is 4 more than the width, find the dimensions.
- (e) A man runs a distance of 12 km at a rate of 8 km per hour. How many minutes does he run? (Hint: notice the units used).

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 4

TOPIC 4: SUMMARY



This summarizes some of the important ideas and concepts to remember.

- A statement that two quantities are equal is called an **equation**. The two quantities are written with an equal sign ($=$) between them. Some equations are true, some are false and some are conditional.
- A **Conditional equation** is an equation which is true for some replacements of the variable and false for others.
- The root or solution of the equation is the number that makes an equation true.
- When using the **addition-subtraction rule** to solve an equation, be sure to add or subtract the same number or expression to both sides.
- When using the **multiplication-division rule** to solve an equation, be sure to multiply both sides by the same non-zero number.
- Use the addition-subtraction rule to **isolate** the variable before using the multiplication-division rule.
- When solving an equation involving **grouping symbols**, first use the distributive law to remove all parentheses.
- Multiply both sides of an equation by an appropriate **factor** to eliminate all fractions and decimals.
- A **Literal equation** is an equation which involves more variables.
- A **formula** is an equation which specifies how a number of variables are related to one another.
- **Changing the subject of an equation** means "re-arranging" the equation so that another variable is the subject.
- When solving an applied or word problem,
 - (1) Read the problem carefully (perhaps several times) and determine what quantity (or quantities) must be found.
 - (2) Represent the unknown quantity (or quantities) using a letter.
 - (3) Make a sketch or a diagram whenever possible.
 - (4) Determine which expressions are equal and write an equation.
 - (5) Solve the equation and state the answer to the problem.
 - (6) Check to see if the answer (or answers) satisfies the conditions of the original problem.

REVISE LESSON 18-24 THEN DO TOPIC TEST 4 IN ASSIGNMENT BOOK 2.

ANSWERS TO PRACTICE EXERCISES 18-24

Practice Exercise 18

1. (a) $m - 4 = 16$
 (b) $9d = 63$
 (c) $n \div 7 = 4$
 (d) $c + 11 = 15$
 (e) $11 + p = 0$
 2. (a) $m = 20$ (b) $d = 7$ (c) $n = 28$ (d) $c = 4$ (e) $p = -11$
 3. c
 4. d
 5. (a) True (b) False (c) False (d) False (e) False
 6. (a) 3 (b) 12 (c) 8 (d) -20 (e) -12
 (f) 2 (g) 8
-

Practice Exercise 19

1. $2(t + 1) = 4$
 $2t + 2 = 4$
 $2t = 2$
 $t = 1$
 Check: $2(1 + 1) = 4$
 $2(2) = 4$
 $4 = 4$
2. $-4(j + 2) = 16$
 $-4j - 8 = 16$
 $-4j = 24$
 $j = -6$
 Check: $-4(-6 + 2) = 16$
 $-4(-4) = 16$
 $16 = 16$
3. $7(3 - v) = -14$
 $21 - 7v = -14$
 $-7v = -35$
 $v = 5$
 Check: $7(3 - 5) = -14$
 $7(-2) = -14$
 $-14 = -14$
4. $-6(8 + f) = 6$
 $-48 - 6f = 6$
 $-6f = 54$
 $f = -9$
 Check: $-6[8 + (-9)] = 6$
 $-6(8 - 9) = 6$
 $-6(-1) = 6$
 $6 = 6$
5. $9(2c - 8) = 0$
 $18c - 72 = 0$
 $18c = 72$
 $c = 4$
 Check: $9[2(4) - 8] = 0$
 $9(8 - 8) = 0$
 $9(0) = 0$
 $0 = 0$

$$\begin{aligned}
 6. \quad (k - 2) - 9 &= 11 \\
 k - 2 &= 20 \\
 k &= 22
 \end{aligned}$$

$$\begin{aligned}
 \text{Check:} \quad (22 - 2) - 9 &= 11 \\
 20 - 9 &= 11 \\
 11 &= 11
 \end{aligned}$$

$$\begin{aligned}
 7. \quad 6x - 2(x + 3) &= 5(2x - 1) \\
 6x - 2x - 6 &= 10x - 5 \\
 4x - 6 &= 10x - 5 \\
 4x - 10x &= -5 + 6 \\
 -6x &= 1 \\
 x &= -\frac{1}{6}
 \end{aligned}$$

$$\begin{aligned}
 \text{Check:} \quad 6\left(-\frac{1}{6}\right) - 2\left(-\frac{1}{6} + 3\right) &= 5\left[2\left(-\frac{1}{6}\right) - 1\right] \\
 -1 - 2\left(\frac{-1+18}{6}\right) &= 5\left[2\left(-\frac{1}{6}\right) - 1\right] \\
 -1 - 2\left(\frac{17}{6}\right) &= 5\left(\frac{-2-6}{6}\right) \\
 -1 - \frac{17}{3} &= 5\left(\frac{-8}{6}\right) \\
 \frac{-3-17}{3} &= \frac{-40}{6} \\
 \frac{-20}{3} &= \frac{-20}{3}
 \end{aligned}$$

$$\begin{aligned}
 8. \quad 3x + 2 - 4(x - 3) &= 2(5x - 4) \\
 3x + 2 - 4x + 12 &= 10x - 8 \\
 3x - 4x - 10x &= -8 - 2 - 12 \\
 -11x &= -22 \\
 x &= 2
 \end{aligned}$$

$$\begin{aligned}
 \text{Check:} \quad 3(2) + 2 - 4(2 - 3) &= 2[5(2) - 4] \\
 6 + 2 - 8 + 12 &= 2(10 - 4) \\
 12 &= 2(6) \\
 12 &= 12
 \end{aligned}$$

Practice Exercise 20

$$\begin{aligned}
 1. \quad (a) \quad 8x + 7 &= 4x - 2 \\
 4x &= -9 \\
 x &= \frac{-9}{4}
 \end{aligned}$$

$$\begin{aligned}
 (c) \quad 5 + 2x &= 11 - x \\
 3x &= 6 \\
 x &= \frac{6}{3} \text{ or } 2
 \end{aligned}$$

$$\begin{aligned}
 (b) \quad 7x + 3 &= 2x + 7 \\
 5x &= 4 \\
 x &= \frac{4}{5}
 \end{aligned}$$

$$\begin{aligned}
 (d) \quad x - 3 &= 5x + 7 \\
 -4x &= 10 \\
 x &= -\frac{10}{4} \text{ or } -\frac{5}{2}
 \end{aligned}$$

2. (a) $3 - x = -12 + 4x$
 $3 - 5 = -12 + 4(5)$
 $-2 \neq 8$
 So, solution is incorrect
- (b) $15 - 3a = 10 - a$
 $15 - 3(1) = 10 - 1$
 $12 \neq 9$
 So, solution is incorrect.
3. (a) $3(x - 1) = 2x + 9$
 $3x - 3 = 2x + 9$
 $x = 12$
- (b) $5x + 1 = -(x + 3)$
 $5x + 1 = -x - 3$
 $6x = -4$
 $x = -\frac{4}{6}$ or $-\frac{2}{3}$
- c. $3y + 2 = 5(y - 6)$
 $3y + 2 = 5y - 30$
 $-2y = -32$
 $y = 16$
-

Practice Exercise 21

1. (a) $x = 15$ (b) $x = 20\frac{1}{2}$
2. $x = -\frac{1}{5}$
3. (a) $x = \frac{6}{5}$ (b) $x = 8$ (c) $x = \frac{12}{7}$
4. (a) $x = 13$ (b) $x = \frac{13}{2}$ (c) $x = -8$
5. (a) $x = \frac{1}{4}$ (b) $x = -\frac{13}{7}$
- (c) $x = 8$ (d) $x = -\frac{5}{11}$
-

Practice Exercise 22

1. (a) $x = y - 3$
 (b) $x = y - m$
 (c) $x = y + 5$
 (d) $x = y + m$
 (e) $x = \frac{y}{8}$
 (f) $x = \frac{y - 5}{2}$
 (g) $x = \frac{y - c}{m}$

$$(h) \quad x = \frac{y + 7}{3}$$

$$(i) \quad x = 2(y - 5)$$

$$(j) \quad x = \frac{4y - 20}{3}$$

$$(k) \quad x = \frac{ty - pt}{w}$$

$$(l) \quad x = \frac{7y - 2}{5}$$

Practice Exercise 23

$$1. \quad (a) \quad T = \frac{l}{PR} \qquad (b) \quad a = \frac{V - u}{t} \qquad (c) \quad r = \frac{C}{2\pi}$$

$$d. \quad x = \frac{y - b}{m} \qquad (e) \quad M = DV$$

$$2. \quad (a) \quad L = \frac{A}{W} \qquad (b) \quad D = \frac{C}{\pi} \qquad (c) \quad h = \frac{V}{\pi r^2}$$

$$(d) \quad b = \frac{2A}{h} \qquad (e) \quad W = \frac{P - 2L}{2}$$

Practice Exercise 24

$$1. \quad (a) \quad 7x = 42 \qquad (b) \quad x + 5 = 37 \qquad (c) \quad 8 - x = 1$$

$$(d) \quad 2x + 4 = 20 \qquad (e) \quad x + 3 = 39$$

$$2. \quad (a) \quad 13 \text{ and } 39$$

$$(b) \quad 84 \text{ and } 86$$

$$(c) \quad \text{Jim is 11 years old and Sam is 44 years old}$$

$$(d) \quad 11 \text{ m and } 15 \text{ m}$$

$$(e) \quad 90 \text{ minutes}$$

END OF TOPIC 4

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